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BEGINNERS' LOGIC

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BEGINNERS' LOGIC

BY

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IN THE PENNSYLVANIA STATE COLLEGE

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PREFACE

During my experience in the teaching of logic and other philosophical subjects in the Pennsylvania State College this book has developed. A textbook rather than a treatise, it does not aim to present new material, but it makes such departures from the traditional order of presentation and selection of material as have been suggested by pedagogical considerations. In each chapter I have endeavored, so far as feasible, to introduce new concepts by means of examples and exercises, instead of abstract discussion. Many of the more recondite problems have been omitted entirely; others have been postponed to the later chapters. The principal exception is Chapter VII, on Symbolic Logic, which has been allowed to occupy what seems to be its natural place at the conclusion of the "deductive" part of the book. Even in this chapter the first few sections are, I believe, understandable by the "meanest intellect"; but the rest of the chapter may well be omitted at the first reading, or in the case of a class studying under a teacher, may be required of only the strongest students. The same is true of certain sections of Chapters X, XI, and XII.

No one reader or student is expected to master all the exercises. Those, however, which have been included in the body of any given chapter should be faithfully studied, as they form an integral part of the exposition. To the advanced student, many of these will doubtless seem obvious and trivial; but this has been deliberately chosen as the lesser of two evils. For the test of merit in an illustration is that it should illustrate. On the other hand, especially in the

exercises appended to Chapters VI and XII, I have purposely included some material which it will be found difficult or impossible to force into any of the recognized forms.

It has been customary to treat the categorical syllogism as the central topic of deductive logic. I have thought it best, however, to make the hypothetical syllogism the basic type of argument, and to derive the rules of immediate inference and of the categorical and disjunctive syllogisms from the principles of the modus ponens and modus tollens. In this way we avoid the use of the difficult and factitious notion of a "distributed term," substituting for it methods of verification which are more nearly akin to the processes of actual thinking; and we also secure greater unity and logical interrelatedness in the exposition of the deductive part of logic than are attainable if the more usual order of topics is followed.

I have not, however, attempted a rigorous mathematical demonstration of the principles of logic, although in a few cases, as in the discussion of conversion and in the justification of the rules of the categorical syllogism, there has been some suggestion of such an attempt. It would, I believe, be possible to present the deductive part of logic in the approved mathematical manner as a system of definitions, postulates, and theorems; but such an attempt falls outside the scope of an elementary textbook.

It has not seemed wise to confuse my readers, most of whom will no doubt be beginners in logic, by a discussion of questions which are subjects of controversy among contemporary schools. Even at the risk of weakening the fabric by leaving a good many loose ends and dropped stitches, I have thought it best to preserve an appearance of neutrality. A brief statement of my own position may, however, be permitted here.

"There are, in general," says Professor C. I. Lewis, "three

types of logical theory: (1) The view which treats logic as formal and at the same time as concerned with the actual modes of right thinking. Traditional logic is of this type. (2) The view which regards logic as concerned with the actual processes of right thinking, and *for that reason* repudiates formalistic conceptions of logic as inadequate. The socalled 'modern logic' illustrates this type. (3) The view which treats logic as formal and renounces all attempts to portray the actual psychological processes which lead to the discovery of truth. Recent mathematical logic—what Mr. Spaulding has called the 'new logic'—belongs to this type." (*Journal of Philosophy,* Vol. XVIII, p. 505.)

Now I do not profess to be a partisan of the "old," the "modern," or the "new" logic. My view is more catholic. It seems to me that logic may properly consist, in part, of forms to which thinking ought to approximate; in part, of forms which neither are nor ought to be those of our actual thinking, but which may nevertheless be used *to test the results* of thinking; and also, in part, of forms which have no relation at all to anybody's thinking except the logician's. In other words, logic is both a science of valid thinking and a pure science of order. In so far as the classical system is a science of order, it is part of a more comprehensive system. In an age which is keen for obvious utility, it is, I believe, good strategy to begin with an appeal to the interest in valid thinking, in the hope of developing in at least some minds an interest in the science of order considered as an end in itself. That is to say, for pedagogical reasons, we should begin with the "old" logic, even if our more fundamental interest is concerned with the mathematical or "new" logic.

Although in most cases it has not been possible to make specific acknowledgments, my obligations to other writers are evident on every page. I have leaned heavily upon contemporary textbooks by Hibben, Creighton, Bode, Sellars,

Turner, Toohey, and, last but not least, L. J. Russell, whose treatment of the syllogism I have found especially helpful. Of the older writers, I have learned most from J. N. Keynes. In the field of mathematical logic I owe much to Couturat, Bertrand Russell, C. I. Lewis, and H. B. Smith.

In conclusion I wish to express my personal indebtedness to Dr. John S. Stahr, late president of Franklin and Marshall College, who was my first teacher in logic; and to Professor A. O. Lovejoy of the Johns Hopkins University, who more than any other man of my acquaintance has seemed to me to exemplify logic in action.

R. H. D.

State College, Pa.
March, 1924.

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BEGINNERS' LOGIC

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CHAPTER I

VALIDITY AND TRUTH

The Definition of Logic.—In the case of logic as in that of most other sciences, the entire first course may properly be considered as an attempt at definition. To *define,* as the etymology of the word itself implies, is to fix the boundaries of a subject, to indicate just what part of the field of possible knowledge it includes. And this cannot be done in a half dozen words. One reason why it is hard to define a science is that the boundaries are not precise. This is especially true of logic. It overlaps epistemology and psychology on the one hand, rhetoric and grammar on the other. Nevertheless the student will probably expect to find a definition of the science at the beginning of his course, and this expectation is not wholly without justification. Before we set out on a journey it is well to have some idea of our probable destination. Remembering, then, that no brief statement can give an adequate impression of the territory which is to be covered, and that any preliminary definition is pretty sure to be only imperfectly understood, we may risk the statement that *logic is the science of valid thinking.*

Logic and Psychology.—The important word in this statement is the adjective "valid." Logic attempts to formu-

late trustworthy ways of arriving at truth; or, to express the same meaning in negative terms, it is a system of rules for the avoidance of fallacy. Not all thinking, then, is the proper subject-matter of logic, but only thinking of the sort which can be trusted to lead us to true conclusions. In other words, logic is not a sub-division of psychology. Logic belongs to the group of sciences called *normative,* while psychology is a science of fact. The meaning of the word normative may be gathered from a consideration of the words "normal," "abnormal," "subnormal," "supernormal," etc. In all these words, the syllable *norm* means *type* or *standard* or *ideal.* Ethics is a normative science because it attempts to discover norms or standards of conduct; aesthetics, because it seeks the norms of beauty. In the same way, logic inquires concerning the norms of correct thinking. The *descriptive* or *factual* sciences, on the other hand—one of which is psychology—simply try to discover facts as they exist and processes as they take place, without raising the question of what ought to be. For example, the astronomer does not ask when the eclipse *ought* to take place; the chemist does not ask what *ought* to be the result if two substances are mixed in a given proportion. Instead each asks simply what will be the course of events, certain conditions having been fulfilled or certain other events having already taken place. To sum up the difference between psychology and logic, the former inquires *how we think,* while the latter seeks to discover *how we ought to think* or how we must think if we are to be sure of attaining the truth.

The Meaning of Validity.—*Exercises.*—In the effort to develop the meaning of the words *valid* and *validity* the tyro will in all likelihood get more help from examples than from abstract discussion. Accordingly, let him tell which of the following arguments are valid, and which are invalid. (For

the present the learner will have to depend upon his own logical "feeling" to enable him to discriminate between valid arguments and those that are untrustworthy. A little later we shall formulate specific rules for this purpose.)

1. Philadelphia is north of Richmond, and Boston is north of Philadelphia. Therefore, Boston is north of Richmond.

2. Lead is heavier than iron; feathers are lighter than lead; therefore feathers are lighter than iron.

3. John is older than Mary, and Rachel is older than John; therefore Rachel is older than Mary.

4. A horse is larger than a dog, and an elephant is larger than a dog; therefore an elephant is larger than a horse.

5. If he goes to the city, he will spend his money. But he has gone to the city. Therefore his money will be spent.

6. If the horse runs away, the carriage will be broken. But the horse will not run away. Therefore the carriage will not be broken.

7. Residents of the District of Columbia are not allowed to vote in the presidential elections. Mr. A is a resident of the District. Therefore he is not allowed to vote for President.

8. A = B and B = C. Therefore A = C.

9. Henry is a cousin of Elizabeth, and Elizabeth is a cousin of William. Therefore Henry is a cousin of William.

10. All quadrupeds are vertebrates. All horses are vertebrates. Therefore all horses are quadrupeds.

11. Silver is heavier than gold, and feathers are heavier than silver. Therefore feathers are heavier than gold.

12. All plane figures are polygons; a cube is a plane figure; therefore a cube is a polygon.

13. If this man is a Parisian, he is a Frenchman. But he is a Frenchman. Therefore he is a Parisian.

14. Premier Lloyd George is a native of London, and all Londoners are Englishmen. Therefore Lloyd George is an Englishman.

15. If this figure is a square, it is a quadrilateral. But it is not a quadrilateral. Therefore it is not a square.

16. All regular polygons are equilaterals; all squares are regular polygons; therefore all squares are equilaterals.

17. All Pennsylvanians are Americans, and all Philadelphians

are Pennsylvanians. Therefore all Philadelphians are Americans.

18. All A is B. All C is B. Therefore all A is C.

19. All horses are bipeds, and all fowls are horses; therefore all fowls are bipeds.

20. All planets are self-luminous and all stars are self-luminous; therefore some stars are planets.

21. No men are infallible and no Europeans are men; therefore no Europeans are infallible.

Validity as Trustworthy Form.—Notice that an argument may be valid even though some or all of its components are false, and invalid even though they are all true. In other words, validity is an affair of the form, and not at all of the subject-matter of an argument. If the reasoning is formally correct, it is said to be valid, regardless of the truth or falsity of the statements which compose it. Consider an argument of the form, "No A is B; all C is A; Therefore no C is B." An argument of this form is manifestly valid, whatever values are given to A, B, and C. Thus if A stands for Pennsylvanians, B for Canadians, and C for Philadelphians, we have as our syllogism,

No Pennsylvanians are Canadians.
All Philadelphians are Pennsylvanians.
∴ No Philadelphians are Canadians.

Not only is this syllogism valid, but all of the statements composing it are evidently true. Suppose, however, we let B represent Americans instead of Canadians, the values of A and C remaining the same as before. Then we have,

No Pennsylvanians are Americans.
All Philadelphians are Pennsylvanians.
∴ No Philadelphians are Americans.

In this syllogism, the first premise and the conclusion are false. Nevertheless the syllogism is valid; for its *form*

remains the same as before, namely, "No A is B; all C is A; therefore no C is B." The difference is solely one of content or subject-matter. In the one case B means Canadians; in the other case it means Americans. In the latter case one of the supporting statements is false, and consequently we have no right to expect a true conclusion; while in the former case we have a right to expect a true conclusion, inasmuch as both of the supporting statements are true. In short, a valid argument is a good machine. If it is fed with good material, it will yield a satisfactory product. But, however good the machine may be, if it is fed with inferior material, the operator has no right to expect anything but an unsatisfactory product.

On the other hand, it is possible for an invalid argument to conclude with a true statement. (Strictly speaking, if the argument is really invalid, there is no conclusion. Yet for convenience' sake, it is customary to call the statement which purports to be the outcome of the reasoning the conclusion.) Of course, as we have seen in the last paragraph, if the argument is invalid, we have no right to look for a true conclusion, even if the premises are true; yet the "conclusion" may happen to be a true statement. For example, should one reason, "All Pennsylvanians are Americans. All Philadelphians are Americans. Therefore, All Philadelphians are Pennsylvanians," the so-called "conclusion," while of course true, does not really *follow* from the premises. That an argument of this form is not valid, becomes clear upon the substitution of other subject-matter. Thus if we put horses in place of Pennsylvanians, quadrupeds of Americans, and sheep of Philadelphians—as we are permitted to do, since validity is wholly independent of subject-matter—we obtain the following: "All horses are quadrupeds. All sheep are quadrupeds. Therefore, All sheep are horses." Altho the premises are true, we cannot

grant the truth of the conclusion. Since an argument of this form cannot be trusted to give a true conclusion even when the premises are true, it cannot be considered a valid argument.

The learner will do well to observe, also, that in some cases an invalid argument may apparently yield a true conclusion, even when the premises are false. Take an example of the same form as the one just discussed: "All quadrilaterals are squares; all rectangles are squares; therefore, all rectangles are quadrilaterals." Here the "conclusion" is true; yet both premises are false, and the syllogism is formally invalid.

In short, while a valid argument is one which can be trusted to lead us from true premises to a true conclusion, truth and validity are nevertheless distinct concepts. The adjectives *true* and *false* are properly applied to propositions, *valid* and *invalid* to arguments. Not only are truth and validity distinct ideas, but neither is an unfailing index of the other. If it is formally correct and if its premises are true, an argument may properly be called *good*. If these conditions are both fulfilled, the conclusion will also be true. However, let us repeat the warning that even if the conclusion is true it does not follow that the argument is valid. To suppose that we can safely reason from the truth of the "conclusion" to the validity of the argument upon which it ostensibly depends is a common error, against which the beginner should be on his guard. For the "conclusion" may be known to be true on grounds other than those alleged in the premises. And just as a jury must disregard everything learned outside the court room and base its verdict entirely upon the evidence as it is presented in open court, so in a valid argument there must be nothing in the conclusion which was not given in the premises by which it is supported, and from which it is said to follow.

Among the examples given in the preceding section, the learner will seek one example, at least, of each of the following cases:

(a) Both premises, or supporting statements, are true, and the argument is valid.

(b) Both premises are false, and the argument is valid.

(c) Both premises are true, and the argument is invalid.

(d) Both premises are false, and the argument is invalid.

(e) One premise is true, one false, and the argument valid.

(f) One premise is true, one false, and the argument invalid.

Logic as an Art.—So far as logic succeeds in discriminating between trustworthy and untrustworthy ways of thinking, it may lay claim to recognition as one of the most important of the arts. For it is of the highest importance that all should think effectively. The student who is beginning the subject should not, however, entertain unwarranted expectations. He will not become an infallible thinker by spending a few weeks—or years—in the study of logic. For logic, as we have just pointed out, is concerned with the formal element only of our thinking. Trustworthy thinking in any subject can, accordingly, be attained in no other way than by study of the subject itself. Nevertheless there are certain principles of method, certain logical forms, which are common to all subjects of investigation; and these it is the special function of logic to discover. The case for logic is much the same as that for the study of English grammar. A knowledge of grammar alone, no matter how complete it may be, will not enable a man to write successfully unless he knows something to write about. For grammar, too, is a formal study. It teaches you how to express your thought, provided you have any thought to be expressed! In another respect logic and grammar are

alike. It is possible for a person who has been extremely fortunate in his early associations—in those whose speech he has heard, and in the books and newspapers which he has read—to speak and write correctly without knowing anything at all about grammar as it is studied in the schools. In the same way it is possible for some fortunate individuals to pick up by imitation, or perhaps to possess by inheritance the forms of correct thinking, without giving any attention to the science of logic. Most of us, however, have not been so fortunate. Little observation of the behavior of men is required to convince us that neither the habit of grammatical speech nor that of logical thinking is very widespread. Grammar is of chief value as a means of becoming aware of bad linguistic habits; and logic as a defence against fallacy.

Logic and Rhetoric.—Considered as an art, logic is nearly allied to rhetoric. The relation between the two is excellently described by Alfred Sidgwick, *Fallacies,* page 15. "Rhetoric is commonly considered as the science of Persuasion (and possibly also of Pleasing) by means of language,—persuasion whether to true or false conclusions; and since persuasion partly depends on showing the person to be persuaded an appearance, whether real or counterfeit, of truth—of absence of fallacy,—the importance to it of a thorough familiarity with Logic is obvious. . . . But though Rhetoric cannot exist without Logic, the latter science can, it seems to me, exist apart from the former. As Mill expressed it, if there were but one rational being in the universe, that being might be a perfect logician: Logic, in this sense, is in fact simpler than Rhetoric, and preliminary to it."

In rhetoric we take the point of view of the advocate; in logic, that of the judge or the jury. Among the ancient Greeks the teachers of rhetoric were called Sophists; and with their long speeches, their eloquence and posturing,

there was probably some color of truth in the accusation of their enemies that they taught their pupils "to make the worse appear the better reason." Their ill fame is preserved in our words *sophistry* and *sophism,* which denote the intentional employment of fallacious arguments. Socrates, on the other hand, was essentially a logician. He was not satisfied with that which sounded well, or which promised to please an audience, but was chiefly concerned with the value of the argument itself. It need hardly be said, however, that the true rhetorician is not one who tries to make black appear white and white black, as the Sophists were accused of doing, but rather one who seeks to commend the bare skeleton of logical discourse by giving it an attractive expression.

The Practical Value of Logic.—Certain benefits may, then, be reasonably expected from the study of logic. (1) It strengthens the demand for definitions. In every argument or investigation each person concerned has the right to demand that there be a complete understanding concerning the precise meaning of all the terms employed. (2) It trains us to look for the assumptions presupposed in our thinking. Next to the errors which arise through the ambiguous use of words, there is no more prolific source of error than the tacit taking for granted of principles which, if dragged to the light of day, would be seen to be untenable. (3) It fosters devotion to the ideal of validity, thus freeing the student from the bondage of prejudice and self-interest, and safeguarding him against the lure of the plausible. (4) It acquaints the learner with the degree of certainty which may reasonably be expected in our thinking. It is just as contrary to the spirit of logic to demand too much in the way of proof, as it is to be too easily satisfied.

Logic a Branch of Philosophy.—In general, the value of logic is the same as that of any other philosophical discipline. Philosophy, it has been said, "bakes no bread." It has no

direct practical value. It does not help one to make a living. Yet, in the larger view, there is nothing quite so practical as philosophy, since there is nothing of which we stand in greater need than the ability to take a dispassionate view of all subjects of inquiry. In theology, in economics, in politics, in international relations, the greatest need of the world is "less heat and more light." And in philosophy, better perhaps than anywhere else, unless it be in the laboratory sciences, the student is challenged to cultivate this spirit of disinterested inquiry. Indeed modern philosophy, like that of the ancient Greeks, is characterized by the effort to carry the scientific method and attitude into fields which have usually been ruled by passion and preconceived opinion. As the "essentially philosophic qualities," Professor Lovejoy enumerates "the sceptical temper, intolerance of obscurity and equivocality, acuteness in logical discrimination, sense of the inter-relatedness and implicativeness of ideas, patience and tenacity in 'following the argument' whithersoever it may lead." [1] This general liberalizing value logic shares with the other philosophic disciplines.

The History of Logic.—The founder of our traditional logic was Aristotle, a Greek thinker who flourished in the fourth century B.C. To fix his date remember that he was the tutor of Alexander the Great. Aristotle, however, did little more than elaborate and systematize the methods of inquiry which his teacher Plato had learned from *his* teacher Socrates. The student is referred to the Platonic dialogues, if he wishes to learn more of the contrast between the "dialectic" of Socrates, out of which logic arose, and the rhetoric of the Sophists. The Aristotelian logic was highly prized by the schoolmen of the Middle Ages, who originated many of the technical terms which are still employed. The mediaeval logicians, however, were chiefly interested in the

[1] *Philosophical Review,* Vol. XXVI, p. 137.

vindication of the doctrines authoritatively taught by the Church. Consequently they restricted their attention almost entirely to the so-called "deductive" logic. That is to say, their dominant aim was to show how, if certain general principles were accepted, other doctrines or principles might be shown to follow logically from them. They were not interested in the discovery of new principles, for they supposed that the first principles of all knowledge were already known, and that they might be found in the Scriptures, in the writings of Dionysius the Areopagite, St. Augustine, and the other Fathers of the Church, and in the books of Aristotle, who was commonly referred to as *"the* Philosopher." At the beginning of the modern period, when the authority of antiquity was discredited, the so-called "inductive" logic, the logic which prescribes methods for the discovery of general laws, was contrasted with the traditional deductive logic, much to the disadvantage of the latter. The inductive logic, the logic of discovery, is frequently associated with the name of Francis Bacon (1561-1626), while the deductive logic, the logic of authority, is associated with the name of Aristotle. This, however, is hardly fair to Aristotle, inasmuch as he recognized the value of both methods. And, in reality, deduction and induction are not distinct methods—still less can they be considered as rival *logics*—but each presupposes the other.

The Plan of this Book.—After the above cursory survey, little attention will be paid in this book to the historical aspects of our science. Little or no space will be given to the discussion of questions which are now at issue between rival schools or tendencies. Many of the more recondite problems will be ignored entirely. The following pages will contain little or nothing that is new, so far as the subject-matter of the science is concerned. An attempt will be made to discuss the science from the standpoint of the beginner

rather than of the teacher or the advanced student. The order in which the topics are presented will be determined by pedagogical considerations. The attempt will be made, so far as feasible, to teach by means of examples rather than of abstract statement. The examples, or "exercises," have been chosen on the ground of their familiarity, and it is hoped that no one will be offended by their trivial character.

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CHAPTER II

THE ELEMENTS OF LOGIC

Types of Argument.—The following are all valid arguments. Notice, however, that no two of them are of the same form.

1. If a man was born in the Keystone State, he reverences the memory of William Penn. This man was born in the Keystone State. Therefore he reverences the memory of William Penn.
2. All good soldiers are courageous. The men of the 28th infantry are good soldiers. Therefore the men of the 28th infantry are courageous.
3. William is either in the Library or in the Chemical Laboratory. He is not in the Laboratory. Therefore he must be in the Library.
4. All Americans are patriots. Therefore no Americans are traitors.
5. Some Americans are Negroes. Therefore some Negroes are Americans.
6. All squares are rectangles. All rectangles are parallelograms. All parallelograms are quadrilaterals. All quadrilaterals are polygons. Therefore all squares are polygons.

The first three of these arguments are examples of different kinds of syllogisms. Notice that each of them consists of three parts,—a conclusion and two supporting statements. The statements which support the conclusion, or from which the conclusion "follows," are called *premises.* Each statement, whether premise or conclusion, is called a *proposition.* If there is but one premise, that is, if the conclusion follows from but one supporting statement, as

in Nos. 4 and 5 above, the argument is called by some logicians an "immediate inference." If the conclusion is supported by more than two premises, that is to say, if the argument consists of more than three propositions, it is a *sorites*. No. 6 is an example of the sorites.

Syllogisms.—To return to a consideration of the three kinds of syllogisms. No. 1 above is a *hypothetical* syllogism; No. 2 is a *categorical* syllogism; and No. 3 is an *alternative* syllogism. Observe that they are all essentially alike except in their first premise. The first, or *major* premise of the first syllogism is a *hypothetical* proposition. The major premise of the second is a *categorical* proposition. That of the third is an *alternative* proposition. Students sometimes confuse the words "proposition" and "syllogism." Remember that a proposition is a single statement; while a syllogism is a complete argument, and is made up of three propositions. Another way, then, of putting the relation between a syllogism and its components is to say that a syllogism takes its name—categorical, hypothetical, or alternative—from its *major* premise. Just what is the difference between the categorical proposition, on the one hand, and the alternative and hypothetical propositions, on the other, will become clear in the light of the examples.

Exercises.—1. Write an original hypothetical proposition. With it as major premise, write a hypothetical syllogism.

2. Write an original alternative syllogism. A categorical syllogism.

3. Classify the following arguments as categorical syllogisms, alternative syllogisms, hypothetical syllogisms, or sorites:

(a) If M is N, P is Q; M is N; therefore P is Q.

(b) A is B if C is D; but A is not B; therefore C is not D.

(c) No X is Y; Some Y is Z; therefore Some Z is not X.

(d) A is either B or C. It is not C. Therefore it must be B.

(e) All cruel men are cowards; No college men are cowards; therefore No college men are cruel.

(f) All wine-growing countries are warm. Spain is a wine-growing country. Therefore Spain is a warm country.

(g) All horses are quadrupeds; all quadrupeds are vertebrates; all vertebrates are animals; all animals are living organisms. Therefore all horses are living organisms.

(h) If he were a gentleman he would not do such a thing. But he has done it. Therefore he is not a gentleman.

(i) X must be at least thirty years of age, because he is a member of the United States Senate.

(j) If a man is eligible for election to the Presidency of the United States, he is a native of the United States. This man is eligible for election to the Presidency. Therefore he is a native of the United States.

(k) Either this man is ignorant of the facts, or he has perjured himself. But he is not ignorant of the facts; therefore he is guilty of perjury.

4. Give the number of hypothetical propositions contained in these arguments. The number of alternative propositions. Of categorical propositions. Notice that each of the hypothetical and alternative syllogisms given above has a categorical minor premise and a categorical conclusion.

The Enthymeme.—Maybe the student has been unable to classify the argument designated (i). At first sight it appears to be an example of "immediate inference." A closer inspection, however, leads to the discovery that it is really a defective syllogism, in which the missing part is the major premise. The argument manifestly presupposes that all United States Senators are at least thirty years of age. Such a defective, or only partially expressed, syllogism is called an *enthymeme.* The word is derived from two Greek words meaning *in* and *mind;* for the unexpressed part may be thought of as *in the mind* of the speaker or writer. Not only the major premise, but in some cases the minor premise, or even the conclusion may thus be unexpressed. In the following enthymemes which part of the syllogism is missing?

(a) You are an Englishman; therefore you should enlist.
(b) All men are mortal; therefore I shall die.
(c) All Frenchmen are polite, and he is a Frenchman.
(d) If a solution turns blue litmus paper red it contains free acid; therefore this solution contains free acid.
(e) Since this form of behavior is socially harmful it ought to be forbidden by law.

Notice that there may be hypothetical as well as categorical enthymemes. Accordingly, if the missing part is the major premise, it may be supplied in the form of a hypothetical or of a categorical proposition, as may be the more convenient. Some logicians speak of enthymemes as of the first, second, or third "order," according as the unexpressed part is, respectively, the major premise, the minor premise, or the conclusion.

In ordinary conversation, and even in written discourse, the complete syllogism is rarely found. For this reason, the logical doctrine of the syllogism is sometimes supposed to be a thing apart from everyday life. The notion of the enthymeme, however, clears up this difficulty. For our spoken and written discussions are full of enthymemes, and the validity of an enthymeme can be tested in no other way than by expressing it in full syllogistic form. When the missing premise is supplied, many a plausible argument is found to be untenable. Indeed, the student will find it a helpful exercise to supply the suppressed premises of all arguments which may be brought to his attention. In this way he will become aware of his own presuppositions or assumptions, as well as of those of others.

The Sorites.—We may now define the sorites as a *compound enthymeme,* or, in other words, as a series of syllogisms each of which is only partially expressed. Consider the following example: "All A is B. All B is C. All C is D. All D is E. Therefore all A is E." If you regard

the first two premises as an enthymeme, what is the missing part? Write the complete syllogism. Next combine the conclusion of this first syllogism with the third premise of the sorites to form another enthymeme, and then write the second complete syllogism. Proceed in this way until every premise has been used. Into how many syllogisms has the sorites been resolved? In the same way expand each of the following into a series of syllogisms:

1. An avaricious man is one who desires more than he possesses;
   A man who desires more than he possesses is discontented;
   A discontented man is unhappy;
   Therefore an avaricious man is unhappy. (Creighton.)
2. All negroes are men; all men are vertebrates; all vertebrates are animals; all animals are mortal; therefore all negroes are mortal. (Bode.)

The Analysis of Propositions.—The three kinds of propositions—the alternative, the hypothetical, and the categorical—have already been distinguished. Let us now discuss the elements of which each of these is composed. First consider two examples of the alternative proposition:

1. Either this gentleman is a Frenchman or else he is a Belgian.
2. William is in the Library, the Gymnasium, or the Laboratory.

Observe that each essential part of an alternative proposition expresses an alternative. For this reason the elements of the alternative proposition are known as *alternants*. How many alternants are there in each of the above propositions?

The hypothetical proposition consists of two parts, which are called the *antecedent* and the *consequent*. The consequent states the result which is affirmed to follow from the condition given in the antecedent. The relation between

antecedent and consequent is therefore analogous to that between cause and effect, or between a sign and the thing signified. In each of these propositions, which is antecedent and which is consequent?

1. If a triangle is equilateral, it is also equiangular.
2. This man will succeed, if he applies himself to his work.
3. If X is a horse, then X is a quadruped.

Notice that, while, as the word itself implies, the antecedent logically precedes the consequent, this order is sometimes reversed. For rhetorical effect the antecedent sometimes comes after the consequent. The student should remember that the clause introduced by "if" or some equivalent word, such as "unless," "until," "when," etc., is the antecedent, whatever be the order in which the clauses actually appear.

As we have seen, the alternative proposition consists of two or more parts; the hypothetical of only two. In the categorical proposition, when it is expressed in strictly logical form, there are always three parts,—the *subject,* the *predicate,* and the *copula.* For example, in the proposition, "All horses are quadrupeds," "horses" is the subject, "quadrupeds" the predicate, and "are" the copula. The copula is always a form of the verb *to be,*—usually *is* or *are,*—and the predicate is a noun (or in some cases an adjective) together with its modifiers, if it has any.

In logic the subject and the predicate of a proposition are known as *terms.* In every categorical proposition there are, accordingly, two and only two terms. The copula is not a term, neither is it a part of the predicate term, but it is the link which connects the subject and the predicate. Observe that a term may be one word or a group of words. In the following exercises indicate the elements of which each proposition is composed.

1. If it rains the ground will be wet.
2. All metals are elements.
3. Unless the solution turns blue litmus red, it does not contain free acid.
4. That man will either be nominated for Governor or for United States Senator.
5. Henry lives in Washington or in Baltimore.
6. All the men of our town are members of the Chamber of Commerce.
7. No men are infallible.
8. Some Americans are Pennsylvanians.
9. Some cows are not Jerseys.

Quantity and Quality of Categorical Propositions.—In which of the categorical propositions given in the preceding section does the subject term denote an entire class? Which of these propositions are *affirmative?* Which are *negative?* If a proposition makes an assertion or a denial concerning an entire class, it is said to be *universal* in quantity. If it does not it is *particular.* For example, "All metals are elements," and "No men are infallible," are universal propositions, because in each case the subject denotes an entire class; while "Some Americans are Pennsylvanians," and "Some cows are not Jerseys," are both particular propositions. The distinction between affirmative and negative propositions is called a difference of *quality.* Do not confuse the quality of a proposition with its *quantity.* If you are asked concerning the quantity of a proposition you are expected to tell whether it is universal or particular; if you are asked about its quality, you are to tell whether it is affirmative or negative. Combining the two notions, we have four types of propositions: the universal affirmative, the universal negative, the particular affirmative, and the particular negative. For convenience these are symbolized, in the order given, by the letters A, E, I, and O. Employing S and P to represent the subject and predicate terms, our

four types of categorical propositions may then be expressed as follows:

A	All S is P	E	No S is P
I	Some S is P	O	Some S is not P

The introductory words, "All," "No," and "Some," do not belong to the subject terms; they are signs of quantity and quality. Other words may be employed instead, for example, "each" or "every" to indicate the universal, and "many," "few," "a few," "not all," etc., to indicate the particular. If no introductory word is employed the proposition is usually understood to be universal. Before deciding that it is universal, however, one should inquire whether the proposition is really to be taken without any exception. If it is really meant to be understood as a universal it may be introduced with "all" or "every." For example, the proposition, "Dogs are carnivorous animals," is obviously equivalent to, "All dogs are carnivorous"; while the proposition, "Dogs are yelping and snarling in our backyard," is hardly to be understood as an assertion about *all* dogs. The former is accordingly universal, the latter particular.

What are the quantity and quality of each of the following? To which of the four types, A, E, I, O, does each belong?

1. Some Frenchmen are Parisians.
2. Not all Frenchmen are Parisians.
3. No horses are carnivorous animals.
4. Some quadrupeds are not horses.
5. Canada thistles are troublesome weeds.
6. All squares are rectangles.
7. Whatever is a triangle is a polygon.
8. Whatever is not a polygon is not a triangle.
9. The demagogue is a menace to democracy.
10. A planet is a body which revolves around the sun.
11. Jealousy is a vice of small minds.

12. Socrates was a citizen of Athens.
13. Oliver Cromwell was not a king.

Observe that Nos. 9, 10, and 11 are propositions of type A; for, in effect, they make affirmations concerning *all demagogues, all planets,* and *all cases of jealousy.* A word of explanation may also be in order concerning the last two. Very likely the beginner will regard them as particular propositions. Strictly speaking, however, they are *individual* or *singular,* because the subject of each denotes but one individual. Thus we have six instead of four types of propositions. However, for most purposes it is convenient to treat the individual proposition as a universal. This procedure is justified theoretically by the convention that a singular term, such as "Socrates" or "Oliver Cromwell," denotes a class,—but a class which contains only one member. And since the class contains but one member, the entire class is of course referred to, and the proposition is therefore universal. One further point may be mentioned. No. 5 is universal; for, while it does not refer to *all thistles,* it refers to all *Canada* thistles, and the subject of the proposition is not *thistles,* but *Canada thistles.* In general, then, we should remember that the subject is not merely the noun but the noun together with its modifiers. Hence the proposition is universal if it refers without exception to the subject class *as it is thus limited.*

Logical Form.—In ordinary discourse propositions of the form, "All A is B," "No A is B," etc., are comparatively rare. It is, however, good practice to transform every proposition brought to one's attention into one or the other of these typical forms. In some cases, to be sure, this will be found exceedingly awkward, and for some logical operations it need not be insisted upon, but for others it is absolutely essential. The proposition, "Flowers grow," may be

written "All flowers are things which grow." "Dogs eat meat," may be changed to "All dogs are animals which eat meat," or "All dogs are carnivora." If the predicate is an adjective, it is usually desirable to supply a noun. Thus, "Horses are useful" becomes "All horses are useful animals." Of course, if the proposition refers to only part of the class denoted by the subject-term, the proper introductory word is "Some" rather than "All." For example, the sentence, "At this very moment men are starving," might take the form, "Some men are beings who at this moment are starving."

The chief points to remember are then three in number: (1) The copula should be distinct from the predicate; (2) the principal word of the predicate should be a noun; and (3) the proposition should begin with "All," "Each," "Every," "No," "Some," "Many," "Few," or some other introductory word which will indicate its quantity. Put the following sentences in logical form, and indicate the quantity and the quality of each by A, E, I, or O.

1. The planets revolve about the sun.
2. Cretans do not speak the truth.
3. Not all men are liars.
4. All is not gold that glitters.
5. Some newspapers did not print the story.
6. Some Americans belong to the colored race.
7. Only seniors are eligible.
8. Only citizens are voters.

Propositions Containing "Only" or "Alone."—The last two exercises may have occasioned some perplexity. "Only citizens are voters," does not mean the same as "All citizens are voters." It means, rather, that "All voters are citizens," or "No non-citizens are voters." In the same way, "Only seniors are eligible," means "All who are eligible are

seniors," or "No non-seniors are eligible." Put the following in logical form:

1. Only lawyers are admitted.
2. Only white men are civilized.
3. Americans alone should be put on guard.
4. Only children and drunken men tell the truth.

Reduction of Categorical Propositions to the Hypothetical Form.—Consider the following equations:

All A is B = Whatever is A is B = If x is A, then x is B.
No A is B = Whatever is A is not B = If x is A, then x is not B.

The form beginning with "Whatever" may be considered as a sort of bridge between the categorical and the hypothetical proposition. It is clear that all universal categorical propositions can thus be reduced to the hypothetical form. Perform this operation in the case of each of the following propositions. (E.g., "If x is a man, x is mortal," is the hypothetical equivalent of the first.)

1. All men are mortal.
2. All metals are elements.
3. Only criminals hate the law.
4. No squares are triangles.
5. No sheep eat meat.
6. All healthy children play.
7. Children alone can appreciate the boredom of nothing-to-do.
8. No good citizens neglect to vote.
9. Socrates was a citizen of Athens. (If x was Socrates, x was a citizen of Athens.)
10. Oliver Cromwell was not a king.
11. A planet is a body which revolves around the sun.
12. The demagogue is a menace to democracy.
13. Honesty is the best policy.

Reduction of Alternative Propositions to the Hypothetical Form.—Not only categorical, but also alternative propositions, may be shown to be equivalent to hypothetical propositions. Indeed the hypothetical proposition may be

conveniently regarded as the generic type, of which the categorical and the alternative are specific modifications. The relation of the alternative and hypothetical forms is illustrated in the following table, in which each proposition of the first column is equivalent to the corresponding proposition of the second column.

Alternative	Hypothetical
1. Either A is B or C is D.	1. If A is not B, C must be D.
2. X is either a knave or a fool.	2. If X is not a knave, he must be a fool.
3. John is either the Secretary or the Treasurer of the Club.	3. If John is not the Secretary of the Club, he must be the Treasurer.
4. Henry resides in Center County or in Clinton County.	4. If Henry does not reside in Center County, he must reside in Clinton County.

Notice that in each of these hypothetical propositions, the antecedent is negative and the consequent affirmative. If we should reverse this arrangement—that is, if we should make the antecedent affirmative and the consequent negative—we should go beyond the information given in the alternative proposition. Thus given the proposition, "Either A is B or C is D," we may interpret it, as we have done above, to be equivalent to "If A is not B, C must be D"; but it is not equivalent to "If A *is* B, C is *not* D," for both alternants may be true. Referring to the other examples given above, X may be both a knave and a fool; and John may be both Secretary and Treasurer. This possibility that both alternatives may be true is expressed in technical terms by saying that the members of the alternative proposition *need not be mutually exclusive.* On the other hand, an alternative proposition is understood to be *exhaustive,* i.e., to enumerate all the alternatives.

Disjunctive Propositions.—An alternative proposition is accordingly not true, unless it enumerates all possible alternatives. "Europeans are either Frenchmen, Germans, or Spaniards," is not true. The alternatives, however, may or may not be mutually exclusive. If the alternatives do in fact exclude each other, the proposition is said to be *disjunctive;* that is to say, the alternatives are completely *disjoined.* Thus some alternative propositions are disjunctive, and some are not. Of the examples given in the preceding section, only the fourth expresses a complete disjunction. Other examples of alternative propositions which are also disjunctive are the following:

This figure is either a square or a triangle.
That man is either American or foreign-born.

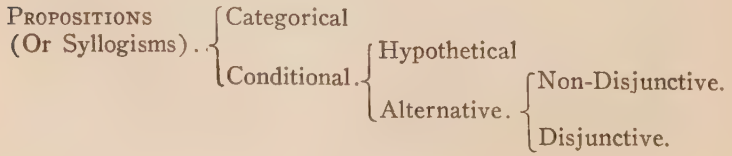
The difference between a disjunctive proposition and a proposition which is merely alternative appears clearly when both types are reduced to the hypothetical form. In the case of the latter type, as we have seen, it is impossible from the *affirmation* of one of the alternatives to draw any trustworthy inference concerning the remaining alternative or alternatives. If, however, the proposition is disjunctive, such an inference is valid. Thus given the proposition, "This figure is either a square or a triangle," we can safely infer, not only, "If it is not a square it is a triangle," but also "If it *is* a square, it is *not* a triangle." For in this proposition we obviously have a complete disjunction.

Exercises.—Which of the following are disjunctive propositions, and which are merely alternative? Reduce each to the hypothetical form.

1. Triangles are either equilateral, isosceles, or scalene.
2. The inhabitants of the valley were either farmers, dairymen or fruit-growers.

3. The man is a native either of New York or of Maryland.
4. Europeans are either Germans, Frenchmen, Spaniards, or Italians.
5. Quadrupeds are either horses, cows, or sheep.
6. Words are either monosyllables, dissyllables, trisyllables, or polysyllables.
7. A is either B or non-B.
8. This animal is either a vertebrate, a biped, or a fowl.

A Classification of Logical Propositions.—Many writers on logic have employed the term *disjunctive* to denote all of the propositions which we have called *alternative;* but then, apparently influenced by the etymology of the word "disjunctive," which suggests the idea of a complete separation, they hesitate between the theory that these propositions do and the theory that they do not give us a complete disjunction. It seems better, accordingly, to use the term *alternative* for the broader class, and reserve the term *disjunctive,* as we have done, for those alternative propositions whose members are known to be mutually exclusive. Many logicians also combine hypothetical and disjunctive (or alternative) propositions under the more general class of *conditional* propositions. We have not had occasion to employ this term, but it may conveniently be included in the classification. All logical propositions which are not categorical are then conditional. The relation of the various kinds of propositions may be exhibited in convenient tabular form as follows. (Since syllogisms take their name from their major premise, this may also be taken as a classification of syllogisms):



Exercises.—1. Write an original hypothetical proposition. An alternative proposition. A categorical proposition.

2. Write an original example of each kind of syllogism.

3. Give examples to illustrate the difference between an enthymeme and immediate inference.

4. Classify the following arguments:

(a) No one who is enslaved by his appetites is free; the sensualist is enslaved by his appetites; therefore no sensualist is free.

(b) If a man is truly noble he is virtuous; but this man is not virtuous; therefore he is not truly noble.

(c) If an airplane is speedier than an express train, it should be used to carry the mail. But it is speedier than an express train; therefore, etc.

(d) All visible bodies shine by their own or by reflected light. The moon does not shine by its own, therefore it must shine by reflected light. (Jevons.)

(e) All ruminant animals have cloven feet. This is a ruminant animal. Therefore it has cloven feet.

(f) Those cultivate the land best who have a personal interest in its improvement, and therefore peasant proprietors are the best cultivators. (Joseph.)

(g) You, as you are old and reverend, should be wise. (Goneril's address to King Lear; quoted by Joseph.)

(h) If light consisted of material particles it would possess momentum. It cannot therefore consist of material particles, for it does not possess momentum. (Jevons.)

(i) If a man is not against me he is for me. This man has shown no sign of hostility. Therefore I shall treat him as a friend.

(j) Because pastry is indigestible it should be eaten, if at all, in very moderate amount.

(k) College men should always be found on the side of public morality, and you are a college man.

(l) Since some Irishmen are Protestants, it follows that some Protestants are Irishmen.

(m) Since these children have the measles they should be quarantined.

(n) Mahomet was a wise lawgiver; for he studied the character of his people.

5. Express the enthymemes as complete syllogisms.

6. Indicate the subject and the predicate of each categorical proposition. The antecedent and the consequent of each hypothetical proposition.

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CHAPTER III

HYPOTHETICAL AND ALTERNATIVE ARGUMENTS

The Structure of the Hypothetical Syllogism.—*Examples.*

1. If the animal is dead it will not move.
   The animal is dead.
   Therefore it does not move.
2. If the animal is dead it will not move.
   The animal does not move.
   Therefore it is dead.
3. If the animal is dead it will not move.
   The animal is not dead.
   Therefore it is moving.
4. If the animal is dead it will not move.
   The animal is moving.
   Therefore it is not dead.

Notice that all four of these hypothetical syllogisms have the same major premise. What is its antecedent? Its consequent? What is the minor premise of each syllogism? In each case does the minor premise relate to the antecedent or to the consequent of the major? If it relates to the antecedent, does it express agreement or disagreement with it? In the syllogisms in which the minor premise relates to the consequent of the major, does it express agreement or disagreement? What is the relation of the conclusion to the antecedent or to the consequent of the major premise?

Which of the foregoing syllogisms are valid? Which invalid? What rule would you suggest for testing the

trustworthiness of hypothetical syllogisms? Using as the major premise of each, "If it rains to-night the ground will be wet in the morning," write four syllogisms like the ones given above. Which of them are valid, and which invalid?

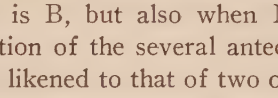
The Rule for Constructing a Valid Syllogism.—There are then four relations in which the minor premise may stand to the major: (1) it may "affirm" or agree with the antecedent; (2) it may "deny" or disagree with the antecedent; (3) it may affirm the consequent; and (4) it may deny the consequent. Of these only the first and fourth produce a valid argument. The second and third are invalid. To sum up, a *hypothetical syllogism is valid if the minor premise affirms the antecedent, or if it denies the consequent, of the major premise.* (It is assumed, of course, that in either form the conclusion follows accordingly.) On the other hand, if the minor premise *denies the antecedent or affirms the consequent,* the argument is fallacious. The valid and invalid moods may be symbolized as follows:

Valid	Invalid
If A is B, C is D.	If A is B, C is D.
But A is B.	But A is not B.
Therefore C is D.	Therefore C is not D.
If A is B, C is D.	If A is B, C is D.
But C is not D.	But C is D.
Therefore A is not B.	Therefore A is B.

The Principle Which Underlies the Rule.—The two valid moods are known respectively as the "modus ponens" and the "modus tollens." In the *modus ponens,* the minor premise affirms the antecedent while the conclusion affirms the consequent of the major premise. In the *modus tollens,* the minor premise denies the consequent, and the conclusion denies the antecedent. In general, then, it is always permissible to argue from the affirmation of the antecedent to

the affirmation of the consequent, or from the negation of the consequent to the negation of the antecedent; but it is not permissible to reverse either procedure.

Why is it fallacious to proceed from a denial of the antecedent to a denial of the consequent, or from an affirmation of the consequent to an affirmation of the antecedent? The reason is to be found in the convention that, while any given antecedent can have but one consequent, a consequent *may* have more than one antecedent. Thus, C may be D, not only when A is B, but also when M is N, or when P is Q. The relation of the several antecedents to the one consequent may be likened to that of two or more tributaries to the river which is formed by their confluence. A simple diagram may help to make the principle clear.



It is evident that the denial of any one of the antecedents need not carry with it the denial of the consequent; since the consequent may be produced by some other antecedent. And if our knowledge is limited to what is actually given in the major premise, we cannot be sure that there are no other antecedents. In the same way the affirmation of the consequent does not necessarily carry with it the affirmation of the given antecedent. The consequent must, indeed, be assumed to have *some* antecedent; but it may be any one of a number of possible antecedents, rather than the one given in the major premise. On the other hand, the affirmation of the antecedent (if the truth of the major premise be granted) implies the affirmation of the consequent, for each antecedent has but one consequent; and the denial of the consequent implies the denial of the antecedent, since it involves the sweeping away of *all* the possible antecedents,

and therefore of the one actually given in the major premise. In short, only the positive method of procedure is valid when the argument moves forward (i.e., from antecedent to consequent), and only the negative procedure is valid when the argument moves backward.

Exercises.—Are the following arguments valid or invalid? Tell whether each valid argument is an example of the *modus ponens,* or of the *modus tollens.* Name the fallacy exemplified by each of the invalid arguments.

1. If this substance is a metal it will be expanded by heat; but it is not a metal; therefore it will not be expanded by heat. Or, it is expanded by heat; therefore it is a metal.

2. If the solution turns blue litmus paper red it contains free acid. The litmus paper remains blue; therefore the solution contains no free acid. Or, the solution contains free acid; therefore the litmus paper will become red.

3. If the figure is a square its angles are all right angles. Now its angles are all right angles; therefore it is a square. Or, it is a square; therefore its angles are all right angles. Or, its angles are not all right angles; therefore it is not a square.

4. A vessel will float if the specific gravity of the material of which it is constructed is less than that of water. The specific gravity of wood is less than that of water; therefore a wooden vessel will float. But the specific gravity of iron is greater than that of water; therefore an iron vessel will sink.

5. If this substance be lead it will weigh more than an equal volume of copper; but it does weigh more than an equal volume of copper, and therefore it must be lead.

6. Unless a certain industrial stock pays regular dividends it is not good business policy to invest one's money in it. But this stock pays dividends semi-annually. Therefore it would be good business policy to buy it. (Notice that "unless" is equivalent to "if . . . not.")

7. Only employees are admitted. No. 121 is an employee. Therefore he will be admitted. (Notice that "only" is equivalent to "if . . . not . . ., then . . . not . . .")

8. Only a genius could have performed this task. This man has performed it. Therefore he is a genius.

9. The bell does not ring unless a train is approaching; the

bell is not ringing; therefore no train is approaching. Or, the bell is ringing; therefore a train is approaching.

10. If a man is patriotic, he votes at every election; this man votes at every election; therefore he is patriotic. Or, this man never votes; therefore he is unpatriotic. Or, he is not patriotic; therefore he does not vote. Or, he is patriotic; therefore he votes.

Cases in Which Antecedent and Consequent are Interchangeable.—If the reader has some knowledge of chemistry, he is likely to object that in the case of the second of the exercises given above all four constructions are valid; for all four yield true conclusions. However, as we saw in the first chapter, even an invalid argument may happen to give a true conclusion. The reason why it seems to be a matter of indifference in this case whether the antecedent or the consequent is affirmed or denied by the minor premise, is that we have additional information. We know more than the major premise tells us of the properties of acid and litmus. We know, in fact, that if the substance contains free acid it will turn blue litmus paper red. It is as if there were two propositions contained in the major premise, so related that when the minor premise denies the antecedent of the one it thereby denies the consequent of the other; and when it affirms the consequent of the one it thereby affirms the antecedent of the other. In short, fallacy is seemingly impossible in this special case because it happens that the antecedent and the consequent of the major premise are interchangeable. Other examples of the interchangeability of antecedent and consequent are the following:

1. If a triangle is equilateral it is equiangular.
   If a triangle is equiangular it is equilateral.
2. If this is water it is H_2O.
   If this is H_2O it is water.

Conversion and Contraposition.—The interchanging of the antecedent and the consequent of a proposition is known technically as *conversion.* Each proposition of a pair such as those discussed above is then the *converse* of the other. However, the simple conversion of a hypothetical proposition is not generally valid. If it were, the denial of the antecedent and the affirmation of the consequent would not be fallacious. While, however, it is invalid to convert a hypothetical proposition, antecedent and consequent may always be interchanged by a process known as *contraposition.* Suppose we have given the proposition, "If this man is a Parisian, he is a Frenchman." By the principle of the *modus tollens* we then know that "If this man is *not* a Frenchman, he is not a Parisian." The second of these propositions is said to be the contrapositive of the first, and *vice versa.* The relation is further illustrated by the following examples:

1. If A is B, C is D.
2. If X is P, X is Q.
3. If it rains the ground will be wet.
4. If this animal is a horse it is a quadruped.

1. If C is not D, A is not B.
2. If X is not Q, X is not P.
3. If the ground is not wet, it has not rained.
4. If this animal is not a quadruped it is not a horse.

In contraposition, then, we first negate the antecedent and the consequent, and then interchange them. Another name for this process is *conversion by negation.* Observe that by *contraposing the contrapositive* we get back to the original proposition. It is then a matter of indifference which is called the original. In other words, hypothetical propositions go in pairs each of which is the contrapositive of the other.

Write the converse and also the contrapositive of each of the following:

1. If this man is industrious he will succeed.
2. If you bring a smile to the glass you meet a smile.
3. A boy will be polite if he is properly trained.

The Equivalence of the Modus Ponens and the Modus Tollens.—Consider again the valid moods of the hypothetical syllogism:

Modus Ponens	Modus Tollens
If A is B, C is D.	If A is B, C is D.
A is B.	C is not D.
Therefore C is D.	Therefore A is not B.

Now let us write syllogisms just like these, except that the major premise of each shall be the contrapositive of the major premise given above. Thus:

If C is not D, A is not B.	If C is not D, A is not B.
A is B.	C is not D.
Therefore C is D.	Therefore A is not B.

Observe that the resulting syllogisms are valid. The minor premise of the first denies or disagrees with the consequent, while the minor premise of the second affirms or agrees with the antecedent of the major premise. The first syllogism, in other words, has now become an example of the *modus tollens,* and the second of the *modus ponens.* In general, then, the *modus ponens* and the *modus tollens* are not essentially different, inasmuch as either can be reduced to the other by the contraposition of its major premise.

Exercises.—1. Contrapose the major premise of each syllogism in the exercises on pp. III.1 and III.2.

2. Show that any argument which was a *modus ponens* is now a *modus tollens,* and *vice versa.*

3. Show that each case of "denying the antecedent" has been changed to an "affirmation of the consequent," and *vice versa.*

4. Observe that propositions beginning with "only" may be

rewritten in two ways, and that each of the resulting propositions is a contrapositive of the other.

The Structure of the Alternative Syllogism.—*Examples.*

1. This figure is a square, a triangle, a circle, or a pentagon.
   It is not a circle.
   Therefore it is a square, a triangle, or a pentagon.
2. That man is a Frenchman, a Spaniard, or an Italian.
   He is not a Spaniard.
   Therefore he is either a Frenchman or an Italian.
3. Henry lives either in Center County or in Clinton County.
   He does not live in Center County.
   Therefore he lives in Clinton County.
4. John is either the Secretary or the Treasurer of the Club.
   He is not the Treasurer.
   Therefore he must be the Secretary.
5. This figure is either a square or a triangle.
   It is a square.
   Therefore it is not a triangle.
6. This man is either a farmer or a Presbyterian.
   He is a Presbyterian.
   Therefore he is not a farmer.
7. X is either A or B.
   It is not A.
   Therefore it is B.
9. X is either A or B.
   It is A.
   Therefore it is not B.
8. X is either A or B.
   It is not B.
   Therefore it is A.
10. X is either A or B.
    It is B.
    Therefore it is not A.

Many other kinds of alternative syllogisms might be constructed. These will be sufficient for our purpose. How many alternants are there in the major premise of each?

In each case, which of the alternants is affirmed or denied by the minor premise? Is the conclusion of each syllogism a categorical or an alternative proposition? Which of these syllogisms seem to you to be valid? Which to be invalid? (Consider only the form, and not the subject-matter.)

Equivalence of the Alternative to the Hypothetical Form.—The reader will recall what was said in the last chapter concerning the reduction of alternative propositions to the hypothetical form. It is evident, then, that any of the above syllogisms can be transformed into hypothetical syllogisms by re-writing the major premise. Consequently we may test the validity of any alternative syllogism by reducing it to the hypothetical form and then applying the rule for the validity of the hypothetical. For example, the major premise of No. 1 may be written, "If this figure is not a circle, it is a square, a triangle, or a pentagon. It is not a circle. Therefore, etc. (as above)." This is valid, since the minor premise agrees with the antecedent of the major.

Note, however, that the major premise of No. 1 might also be written, "If this figure is not a square, etc."; "If . . . not a triangle, etc."; or "If . . . not a pentagon, etc." In the same manner, corresponding to the major premise of No. 3, we have two hypotheticals: "If Henry does not live in Center County he lives in Clinton County," and "If Henry does not live in Clinton County he lives in Center County." It is evident, however, from the principle of the hypothetical syllogism that each of these propositions is an implicate of the other; and whichever we take as major premise, the syllogism is valid. Why? Show that Nos. 2 and 4 are also valid.

The Fallacy of Imperfect Disjunction.—Is No. 5 a valid syllogism? We shall probably agree that it is good reasoning. But suppose we test it as we have tested the others.

The major premise is equivalent to the proposition, "If this figure is not a square it is a triangle." The minor premise then denies the antecedent, and consequently the syllogism is formally invalid. The reasoning, however, is felt to be satisfactory, because we know more about squares and triangles than is contained in the major premise as we have interpreted it. We know that if the figure *is* a square it is *not* a triangle. In other words, the major premise is not merely alternative; it is *disjunctive,* inasmuch as the alternatives—square and triangle—are mutually exclusive.[1] The next syllogism, No. 6, altho in form the same as No. 5, is manifestly fallacious. The major premise is not disjunctive. The alternatives—farmer and Presbyterian—are not mutually exclusive, for it is clear that the man may be both a farmer and a Presbyterian. The technical name for this type of error is the *fallacy of imperfect disjunction.*

In general, then, beware of an alternative syllogism in which the minor premise *affirms* one of the alternants of the major. Such a syllogism is invalid except in the special case that the major premise is really a disjunctive proposition. Thus Nos. 9 and 10 are invalid; for we have no assurance that X may not be A and B at the same time. On the other hand, Nos. 7 and 8 are formally valid; as is true of *any* alternative syllogism in which the minor premise *denies* one of the alternants (provided, of course, that the conclusion is properly drawn).

The Fallacy of Incomplete Enumeration.—However, while all syllogisms in which the minor premise denies one of the alternants are *formally* correct, there is grave danger, in their case, of falling into what is called a "material fallacy," or an error which concerns the subject-matter of one's reasoning. In other words, the major premise is likely to be *untrue,* for it is only in rare cases that we can

[1] See p. 25f.

be perfectly sure that it enumerates all the possible alternatives. The major premise asserts, let us say, that a certain man "is a Frenchman, a Spaniard, or an Italian." But not all nationalities have been enumerated. He may, perhaps, be an Austrian, a Pole, or a Dane. In the use of the type of alternative syllogism in which the minor premise denies, we must, then, pay particular attention to a precept which applies as well to all reasoning. *Be sure,* not only that your argument is formally correct, but also *that your premises are true.*

To sum up our discussion of the alternative syllogism, if the minor premise *affirms* one (or more) of the alternants of the major premise, the conclusion should *deny* the remaining alternant (or alternants); while if the minor premise *denies,* the conclusion should *affirm.* The former mode of argument is not sound, however, unless the major premise is genuinely disjunctive; and the latter mode should not be employed unless it is absolutely certain that the major premise enumerates all the possibilities.

Exercises.—Reduce the following arguments to the hypothetical form, and tell whether each is valid or invalid.

1. Either A is B, or C is D.
   But A is not B.
   Therefore C is D.

2. Either A is B, or C is D.
   But A is B.
   Therefore C is not D.

3. Either A is B, or C is D.
   Now C is D.
   Therefore A is not B.

4. Either A is B, or C is D.
   Now C is not D.
   Therefore A is B.

5. Roses are either red or white.
   This rose is not red.
   Therefore it is white.

6. Roses are either red or white.
   This rose is white.
   Therefore it is not red.

7. Every one in America lives either east or west of the Mississippi. This man lives east of that river. Therefore he does not live west of it.

8. The man you saw yesterday is either an automobile mechanic or a blacksmith. He is a blacksmith; therefore he is not an automobile mechanic. Or, he is an automobile mechanic; therefore he is not a blacksmith.

The Dilemma.—Consider the following arguments:

1. If he goes to the mountains he will spend his money, and if he goes to the seashore he will spend his money; but he will either go to the seashore or to the mountains; therefore his money will surely be spent.

2. If you go out on the icy side-walks you are in danger of breaking your neck; and if you stay indoors you are likely to get sick for want of pure air. But you must either go out or stay in. Therefore you must run the risk of a broken neck or of illness.

3. If this man were wise he would not speak irreverently of Scripture in jest; and if he were good, he would not do so in earnest. But he does it either in jest or in earnest; therefore he is either not wise or not good. (Whately.)

Notice the form of the major premise in each of these arguments. Divide each major premise into two hypothetical propositions. In which do both hypothetical propositions have the same consequent? What sort of proposition is the minor premise, and what is its relation to the major premise? What sort of proposition is the conclusion of each argument?

Arguments of this general form are called *dilemmas*. The dilemma is then a combination of hypothetical and alternative, and in some cases also, categorical propositions. The major premise always consists of at least two hypothetical propositions. When it contains three, the argument may be called a *trilemma*. The minor premise is usually an alternative proposition. The conclusion may be either alternative or categorical. The function of the minor premise, when alternative, is to restrict our range of choice to the alterna-

tives specified by the antecedents or by the consequents of the major premise. There are two kinds of dilemma, corresponding to the *modus ponens* and the *modus tollens* of the hypothetical syllogism. If the minor premise alternatively *affirms* the antecedents of the major, and the conclusion alternatively affirms the consequents, the dilemma is said to be *constructive*. If, on the other hand, the minor premise alternatively denies the consequents of the major premise, and the conclusion alternatively denies the antecedents, the dilemma is then said to be *destructive*.

As a combination of the hypothetical and the alternative types of argument, the dilemma is naturally exposed to most of the perils of both. The chief danger of fallacy, however, as in the case of the alternative syllogism, lies in the difficulty of finding a premise which will exhaustively enumerate the alternatives.

Exercises.—Classify the following arguments. Which are valid? Explain.

1. If the moon has no atmosphere, there cannot be any twilight; but it has no atmosphere; therefore there is no twilight on the surface of the moon.

2. If Aristotle is right, slavery is a proper form of society; but slavery is not a proper form of society; therefore Aristotle is not right.

3. It is either raining or not raining; it is not raining; therefore it is raining. (Bode.)

4. If the door were locked, the horse would not be stolen; but the horse is not stolen; therefore the door must have been locked.

5. No honest man can advocate a change in the creed of his church; for he must either believe it or not believe it; and if he believes it he cannot honestly help to change it, while if he does not believe it he cannot honestly belong to the church at all.

6. If transportation is not felt as a severe punishment, it is in itself ill-suited to the prevention of crime; if it is so felt,

much of its severity is wasted, from its taking place at too great a distance to affect the feeling, or even to come to the knowledge, of most of those whom it is designed to deter; but one or the other of these must be the case; therefore transportation is not calculated to answer the purpose of preventing crime.

7. If every ghost story is to be believed, we must accept the general standpoint of the spiritualists; but we cannot accept their general standpoint; therefore we cannot believe ghost stories. (Aikins.)

8. If peace at any price is desirable, war is an evil; and as war is confessedly an evil, peace at any price is desirable. (Jevons.)

9. In order to move, a body must move either in the place where it is, or in the place where it is not. But it cannot move in the place where it is, since that place is already occupied. Neither can it move in the place where it is not. Motion is therefore impossible.

10. If the earth were of equal density throughout, it would be about 2½ times as dense as water; but it is about 5½ times as dense; therefore the earth must be of unequal density. (Jevons.)

11. If education is popular, compulsion is unnecessary; if unpopular, compulsion will not be tolerated.

12. If a country is mountainous, it will have a large rainfall; this country is mountainous; therefore, etc.

13. The applicant is a graduate either in the arts or in science. I am informed, however, that he is not a graduate in the arts; therefore he must be a graduate in science. Or, he is a graduate in the arts; therefore he is not a graduate in science.

REFERENCES

Creighton, *An Introductory Logic,* Chap. XI.
Bode, *Outline of Logic,* Chap. VII.
For advanced study:
Lodge, *Modern Logic,* Chapters XI and XII.
Bradley, *Principles of Logic,* 2nd ed., Vol. I, Chap. II; also Vol. II, Note X.

CHAPTER IV

IMMEDIATE INFERENCE

Opposition and Equivalence.—*Examples.*

1. All men are honest.
2. No men are honest.
3. Some men are honest.
4. Some men are not honest.
5. Some men are dishonest.
6. Some men are not dishonest.
7. No men are dishonest.
8. All men are dishonest.
9. No honest beings are men.
10. All honest beings are men.
11. Some honest beings are men.
12. Some honest beings are not men.
13. All A is B.
14. No A is B.
15. Some A is B.
16. Some A is not B.
17. No A is non-B.
18. All A is non-B.
19. No B is A.
20. No non-B is non-A.

Which of the foregoing propositions expresses the same meaning as No. 1? As No. 2? No. 3? No. 4? No. 13? No. 14? Which express meanings opposed to these? (Do not attempt to give reasons for your answers, but for the present rely solely upon your logical "feeling.") Write one proposition which seems to you to express the same meaning as, and two which seem to be opposed in meaning to, each of the following:

1. No planets are inhabited.
2. No Cretans speak the truth.
3. Some Pennsylvanians are farmers.
4. Some Europeans are Roman Catholics.

Contraries and Contradictories.—Consider the following propositions:

1. All men are fallible. 2. No men are fallible.
3. Some men are fallible. 4. Some men are not fallible.

Notice that all of these have the same subject, and also the same predicate. Using the letters A, E, I, O, indicate the quantity and quality of each. Which of them agree in quantity? In quality? Which differ in quantity? In quality? In both quantity and quality?

Propositions which have the same subject and predicate, but differ in both quantity and quality, are called *contradictories.*

Propositions which have the same subject and predicate, but differ in quality while agreeing in quantity, are called *contraries* if their quantity is universal; and *sub-contraries* if it is particular. There are accordingly a pair of contraries, a pair of sub-contraries, and two pairs of contradictories. The contradictories are diametrically opposed. The opposition of contraries and of sub-contraries is not so thoroughgoing. If one of two contradictories is true, the other must be false; if one is false, the other must be true; On the other hand, both contraries may be false, but they cannot both be true; while both sub-contraries may be true, but they cannot both be false.

Which of the four propositions given above are contradictories? Which are contraries? Which are sub-contraries? Pick out pairs of contraries, sub-contraries, and contradictories in the group of propositions given in the preceding section. Write the contrary and also the contradictory of "All horses are quadrupeds." Of "All dogs eat meat."

Subalterns.—*The Meaning of "Some."*—Thus far no name has been given to the relation between propositions which have the same subject and predicate, but differ in quantity while agreeing in quality. This relation may con-

veniently be called that of *principal* and *subaltern*. Thus if we take, "All A is B," as the principal proposition, "Some A is B" is its subaltern; while if we take "No A is B" as principal, "Some A is not B" is its subaltern. In other words, the subaltern of an A proposition is an I proposition, and that of an E proposition is an O proposition.

In order to understand the relation between principal and subaltern, one must know the precise sense in which the word *some* is employed in logic. This may be made clear by the use of an example. If a speaker should say, "Some persons in this room are honest," would his auditors have a right to feel insulted? It is probable that many would infer that since "Some are . . ." it must also be true that "Some are not . . ." This inference is not logically justified, however. For, while ordinary usage may imply that *some* means "only some," or "some, but not all," in logic *some* is always understood to mean "some at least," that is to say, "some *and perhaps all.*" Thus, "Some planets are inhabited," need not imply that "Some planets are *not* inhabited"; for the statement that *some are inhabited* is perfectly compatible with the statement that *all are inhabited.*

Accordingly, as the word *some* is used in logic, we can always infer the truth of the subaltern from that of its principal, but we cannot infer the truth of the principal from that of the subaltern. For example, if it be true that "All men are mortal," it must certainly be true that "*Some* men are mortal." On the other hand, given the truth of the proposition, "Some of the trees in this wood-lot are oaks," we have no right to infer either the truth or the falsity of the proposition, "All the trees of the wood-lot are oaks." It may be that all are, and it may be that some are not. The point to be observed is that our proposition has given certain information about *some* of the trees; and the proper attitude toward the rest of the trees is one of "logical caution,"

happily illustrated by the hackneyed legal phrase, "further deponent sayeth not."

The Square of Opposition.—The relations obtaining among contraries, contradictories, and subalterns constitute the so-called "square of opposition." This is represented by the accompanying diagram. A, E, I, and O, stand, as usual, for the four combinations of quantity and quality. All four propositions have the same subject and predicate, which are symbolized by S and P. The horizontal lines

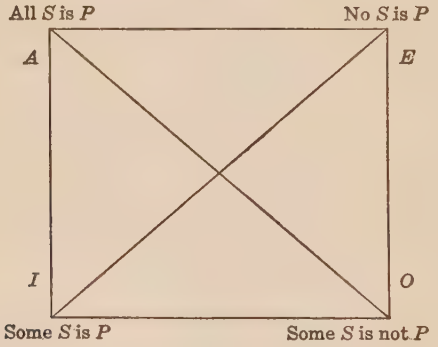


FIG. 1.—THE SQUARE OF OPPOSITION.

indicate the relation of *contrariety,* the oblique lines that of *contradiction,* and the vertical lines that of *subalternation.*

Let us now sum up the possibilities of inference as illustrated by the diagram. Since it is impossible for both contradictories to be true, it is always safe to argue from the truth of either of them to the falsity of the other; and since it is impossible for both to be false, it is safe to argue from the falsity of either to the truth of the other. In other words, along the *oblique* lines of the diagram, inference may proceed in either direction from truth to falsity or from falsity to truth. The case is different, however, when we

turn to the *horizontal* lines. Contraries cannot both be true, and it is accordingly safe to argue from the truth of one to the falsity of the other; but since they may both be false, we cannot argue from falsity to truth. Exactly the reverse is true of the sub-contraries. They may both be true, but cannot both be false. Hence we can argue from the falsity of one of them to the truth of the other, but not from truth to falsity. The case of the *vertical* lines is different still. Here, granted the truth of the principal, we can argue *downward* to the truth of the subaltern, but we cannot argue *upward* from the truth of the subaltern to that of its principal; if, however, we are given the falsity of the subaltern we can argue *upward* to the falsity of the principal, but given the falsity of the principal, we cannot argue *downward* to that of the subaltern.

If, then, we know that A is true, we have the right to infer that E is false, that O is false, and that I is true. If, however, A is false, the only safe inference is that O is true. In this case we remain in doubt concerning the truth or falsity of E and I. On the other hand, if we know that I is true, our only safe inference is to the falsity of E. So far as our information goes, A and O may be either true or false; we cannot tell. If, however, I is false, we know that A is also false, and that each of the others is true. Suppose E is false, what will you know about the others? Suppose E is true? Work out the relations also for O.

Exercises on Opposition.—Arrange the following propositions in pairs as contradictories, as contraries, or subcontraries, or as principal and subaltern.

1. No squares are circles.
2. All squares are circles.
3. Some squares are circles.
4. Some squares are not circles.
5. Some soldiers are not brave.
6. All soldiers are brave.

7. Some soldiers are brave.
8. No soldiers are brave.
9. All M is N.
10. Some M is N.
11. No M is N.
12. Some M is not N.

The Opposition of Terms.—The technical terms *contrary* and *contradictory* are reserved, as defined above, to name such propositions as have the same subject and predicate. It is obvious, however, that we can write propositions having the force of a contrary or a contradictory by using other words. The simplest way to do this is to change the predicate term. Thus, given the proposition, "All sheep are ruminants," the proposition known technically as the contrary is, "No sheep are ruminants." But "All sheep are non-ruminants," would have the force of a contrary. In like manner, while the technical contradictory is, "Some sheep are not ruminants," the proposition, "Some sheep are non-ruminants," has the force of a contradictory of the original proposition. In other words, there are two ways in which we may express opposition to any given proposition. We may change the quality, or in the case of the contradictory, both quantity and quality, of the proposition; or we may change the quality of the predicate term. From the common-sense principle that "two negatives make an affirmative," it is evident that if we change the quality of a proposition, and also the quality of the predicate term (everything else remaining the same), we get back in meaning to the original statement; for these two changes exactly offset each other. For example, "No sheep are non-ruminants," is exactly equivalent to, "All sheep are ruminants." A proposition related to another in this way is called its obverse. Indeed, each may be said to be the obverse of the other. Before going on, however, to discuss *obversion* and other forms of immediate inference, it will be well to turn aside for a minute to inquire concerning the opposition of terms.

Contradictory Terms.—We have seen that propositions may be opposed as contradictories, or as contraries. A corresponding distinction holds of the opposition of terms. A and non-A, B and non-B, are pairs of contradictory terms. They denote alternatives which are both *co-exhaustive* and *mutually exclusive*. They are co-exhaustive, because they share the universe between them; they are mutually exclusive because they have nothing in common. In other words, there are no other alternatives, and the given alternatives do not overlap. For whatever is not included in A must be included in non-A, and nothing can be both A and non-A at the same time.

Contrary Terms.—On the other hand, terms which are mutually exclusive but not co-exhaustive are known as *contraries*. Thus *black* and *white*, if we speak strictly, are not contradictory terms. For a thing may be neither black nor white; it may be blue or gray. Nevertheless, black and white are contraries, since the same thing cannot be (entirely) black and white at the same time. Recalling our discussion of alternative propositions, we may say that in the case of contrary, as well as in that of contradictory terms, there must be a perfect disjunction. If the terms are contrary without being contradictory, the possibilities are not completely enumerated. If the terms are genuinely contradictory there is no additional alternative. For example, *less* and *greater* are contraries but not contradictories; for there is a third possibility, namely, that the quantities in question may be *equal*. The contradictory of *less* is *not-less*, which includes both greater and equal; while the contradictory of *greater* is *not-greater*, which includes both less and equal.

The Universe of Discourse.—We have said that contradictory terms are co-exhaustive. They divide the universe between them. A and non-A between them include

all things. *Honest men* and *dishonest men, isosceles triangle* and *scalene triangle,* do not, however, exhaust the universe; for there are things which are not men or triangles at all. These are, accordingly, contraries rather than contradictories. Nevertheless, if we agree to limit our consideration to the class *man, honest man* may be treated as the contradictory of *dishonest man;* and if we limit our consideration to the class *triangle, isosceles triangle* may be regarded as the contradictory of scalene triangle. The class within which our thought moves in any given case is called the "universe of discourse." If we are talking about men, then the class *man* is the universe of discourse. If we are talking about triangles, the class *triangle* is the universe of discourse. The universe to which our terms refer may accordingly be larger or smaller, more inclusive or less inclusive; and pairs of terms which are merely contraries within one universe of discourse may be contradictories within a more limited universe. Thus *true* and *false* are only contraries if we include all entities in our universe, for there are some things, as for example blades of grass and mud-pies, to which these adjectives do not properly refer. If, however, we limit our universe to the class *proposition* or *judgment, true* and *false* are genuine contradictories.

Exercises.—1. For each of the following pairs define a "universe of discourse" in which the terms are contradictories: right, oblique; isosceles, scalene; male, female; moral, immoral; White, Colored; Democrat, Republican; negotiable, non-negotiable; Senator, Representative; participant, spectator.

2. What is the contradictory of each of the following terms? In each case specify the universe of discourse.

(a) White man.
(b) College Freshman.
(c) Non-S.
(d) Animal.
(e) Newfoundland dog.
(f) P.
(g) Mars.
(h) Mr. Pickwick.
(i) Patriot.

3. If the universe of discourse is taken without any restriction whatsoever, which of these terms are contradictories, and which are only contraries? Agreeable, disagreeable; red, blue; man, not-man; light, heavy; Baldwin apple, that which is not a Baldwin apple; Baldwin apple, apple which is not Baldwin; M, non-M; horse, not-horse; quadruped, biped; monosyllable, polysyllable; monosyllable, word which is not monosyllabic; patriot, traitor; alien, citizen; colored, colorless; long, short; triangle, non-triangle.

Obversion.—After this brief detour we are ready to take up again the topic of obversion. We have compared the obverse with the contrary. We have seen that changing the quality of a proposition, while retaining the identical subject and predicate, produces the contrary; if, however, we also change the quality of the predicate term, the two changes offset each other, and the resulting proposition has the same meaning as the original. A proposition thus formed is an *obverse*. If, then, we have two propositions one of which is affirmative and the other negative, but which express the same meaning, either of these is the obverse of the other. In other words, obversion is the process by which we change the quality of a proposition without changing its meaning. All categorical propositions may be obverted. Consider the following examples:

Obvertend (i.e., the proposition to be obverted)	Obverse
1. All men are honest.	No men are dishonest.
2. No Americans are traitors.	All Americans are patriots.
3. Some Americans are patriots.	Some Americans are not traitors.
4. Some patriots are non-Americans.	Some patriots are not Americans.
5. All S is P.	No S is non-P.
6. Some S is P.	Some S is not non-P.

Observe that if the obvertend is affirmative, the obverse is negative, and if the obvertend is negative, the obverse is

affirmative; but that this change is compensated for by a change in the predicate. *Do not make the mistake of attempting to obvert by changing the subject instead of the predicate.*

It is not necessary in all cases to put the proposition in strictly logical form before obverting. Thus if "All dogs eat meat," it follows that "No dogs refuse meat." Furthermore in changing the quality of the predicate, it is not always necessary to insist upon replacing it by its contradictory. In many cases the contrary will suffice. Yet, from, "No triangles are honest," we cannot infer that "All triangles are dishonest." And, in general, unless the contradictory of the predicate is used—which in this case would give us as obverse, "All triangles are things which are not honest,"—we must be sure that the attribute denoted by the predicate is one which may be significantly applied to the subject.

Exercises.—Which of the propositions given in the second column may be obtained by obversion, and which are contraries or subcontraries of those given in the first column:

1. All automobiles are expensive.	1. No automobiles are expensive.
2. All automobiles are inexpensive.	2. No automobiles are inexpensive.
3. Some vertebrates are quadrupeds.	3. Some vertebrates are not non-quadrupeds.
4. Some vertebrates are not quadrupeds.	4. Some vertebrates are non-quadrupeds.
5. No men are infallible.	5. All men are fallible.
6. No men are fallible.	6. All men are infallible.

Conversion and Contraposition.—Two other ways of varying the form of a proposition without changing its meaning are *conversion* and *contraposition*. The following are examples of conversion:

Convertend	Converse
1. No Americans are traitors.	1. No traitors are Americans.
2. No lions are elephants.	2. No elephants are lions.
3. Some negroes are Americans.	3. Some Americans are negroes.
4. Some planets are inhabited.	4. Some inhabited bodies are planets.
5. No S is P.	5. No P is S.
6. Some S is P.	6. Some P is S.

What is the subject and what the predicate of each of the foregoing propositions? (Remember that "No" and "Some" do not belong to the subject term, but are simply signs of quantity and quality.) What is the relation between the subject of each convertend and the predicate of its converse? Between the predicate of the convertend and the subject of the converse? How do the convertend and the converse compare in regard to quantity and quality? The rule for conversion may now be stated provisionally as follows:

To convert a proposition, interchange subject and predicate, but make no change in quantity or in quality.

The requirement that there is to be no change in the quantity or quality of the proposition is insisted upon only in the case of what is called *simple conversion.* In *conversion by limitation* the quantity is changed from universal to particular; and in *contraposition,* which some writers consider a sub-form of conversion, there may be a change of quality. For the present, however, let us limit our study to simple conversion. With the rule just given in mind, write the simple converse of the following:

1. No M is N.
2. Some P is S.
3. Some Pennsylvanians are farmers.
4. No parallelograms are triangles.

Let us now consider several examples of contraposition.

Contraponend	Contrapositives
1. Some plane figures are not triangles.	1. Some non-triangles are plane figures; or, Some non-triangles are not things which are not plane figures.
2. All Americans are patriots.	2. No non-patriots are Americans; or, All non-patriots are non-American.
3. All dogs are carnivorous.	3. No non-carnivorous animals are dogs; or, All non-carnivorous animals are non-dogs.
4. Some planets are not inhabited.	4. Some uninhabited bodies are planets; or, some uninhabited bodies are not non-planets.
5. All S is P.	5. No non-P is S; or, All non-P is non-S.
6. Some S is not P.	6. Some non-P is S; or, Some non-P is not non-S.

Observe that each contraponend has two contrapositives, and that each of these is the obverse of the other. Notice, too, the relation of the subject of each contrapositive to the predicate of its contraponend. We thus arrive at the rule for contraposition:

Take the contradictory of the old predicate as the new subject, and either the old subject or its contradictory as the new predicate, changing the quality of the proposition or keeping it unchanged as may be necessary to avoid a change in the meaning of the given proposition. In other words, *negate* and then *interchange* the terms.

Since this operation manifestly combines the operations of obversion and conversion, the rule for contraposition may also be stated thus:

To find a contrapositive of any proposition, *first obvert it and then convert the obverse.*

For example, let "All men are mortal," be the given proposition. We first obvert it, obtaining as the result, "No men are immortal." The next step is to convert this result, which gives us as the contrapositive, "No immortal beings are men." This is the *partial* contrapositive. To obtain the *full* contrapositive, we may next obvert the partial contrapositive. Returning to our example, if the original proposition, or the *contraponend,* is "All men are mortal," the partial contrapositive is, "No immortals are men," and the full contrapositive, "All immortal beings are beings that are not men."

Observe that the original proposition and the full contrapositive are of the same quality, while the quality of the partial contrapositive is different. This is another application of the principle that *two negatives make an affirmative.* The essence of contraposition is that before converting the proposition we negate the predicate term. But to retain the meaning of the proposition it is necessary to compensate for the negation of the predicate, either (as in the partial contrapositive) by changing the quality of the proposition, or (as in the full contrapositive) by negating the subject term.

As another example, take the particular negative proposition, "Some men are not soldiers." The contradictory of *soldiers* is *non-soldiers,* or "beings that are not soldiers." The partial contrapositive is therefore "Some beings that are not soldiers are men." But the contradictory of *men,* the subject term of the contraponend, is *non-men,* or "beings that are not men." The full contrapositive is, accordingly, "Some non-soldiers are not non-men," or, "Some beings that are not soldiers are not beings that are not men."

Exercises.—1. Write both contrapositives of the following:

(a) All A is B. (The partial contrapositive is, "No non-B is A," and the full contrapositive, "All non-B is non-A.)

(b) Some A is not B. (The two contrapositives are, respectively, "some non-B is A," and "Some non-B is not non-A.")

(c) All grains are grasses.

(d) Some animals are not wild.

(e) Some metals are not useful.

(f) All metals are elements.

(g) All S is P.

(h) Some S is not P.

2. What is the relation (obverse, converse, contrapositive) of each of these propositions to each of the others?

(a) All knowledge is useful.

(b) No useless thing is knowledge.

(c) No knowledge is useless.

(d) All useless things are things which are not knowledge.

Further Remarks on Propositions Containing "Only" or "Alone."—Let us recall our discussion of "logical form." The student has discovered, no doubt, that there are often several ways of putting the same proposition in logical form. For example, the same thought may be expressed either affirmatively or negatively. Again, the order of the terms (subject and predicate) may be varied. We now see that the various logical propositions which are thus found to be equivalent to a given proposition, are related to each other by obversion, conversion, or contraposition. In particular, we remember that, "Only citizens are voters," is equivalent either to, "All voters are citizens," or to, "No non-citizens are voters;" [1] and each of these is now seen to be the partial contrapositive of the other.

Put the following in logical form in two different ways:

[1] See p. 22.

1. Only the brave deserve the fair.
2. Only his most intimate friends were invited.
3. Only natives of the United States are eligible.
4. Mathematicians alone are able to understand that demonstration.

Errors in Obversion and Conversion.—Consider the following propositions: 1. All S is P. 2. No non-S is P. 3. No S is non-P. Which of these is the obverse of the first? Which is true if No. 1 is true? It may seem at first sight that both Nos. 2 and 3 would be valid inferences from No. 1. For example, if one were to assert that "All righteous persons are happy," another might be tempted to interpret this as equivalent to "No unrighteous persons are happy." This would, however, be unjustified, as appears even more clearly if we try to pass from the statement that "All horses are quadrupeds," to the statement that "No non-horses are quadrupeds." The error just exemplified consists in attempting to compensate for the change in the quality of the proposition by changing the quality of the *subject* term. Remember that in obversion we must change the quality of the *predicate*. The attempt to obvert by manipulating the subject has been called "false obversion." It is analogous to "denial of the antecedent" in the case of the hypothetical syllogism. This is evident if we reduce both propositions to the hypothetical form. It will be found that "All S is P," is equivalent to, "If x is S, then x is P," while "No non-S is P," is equivalent to, "If x is not S, then x is not P." These are manifestly not equivalent to each other. If they were, a minor premise which denies the antecedent would produce a valid syllogism. On the other hand, the genuine obverse, "No S is non-P," is equivalent to the hypothetical proposition, "If x is S, then x is not non-P." Remembering that two negatives make an affirmative, it is clear that this expresses the same meaning

as, "If x is S, then x *is* P," which, as we have seen, is the hypothetical equivalent of the original proposition.

There is danger of similar errors in conversion. The reader may have wondered why no attempt has been made to convert the universal affirmative and the particular negative propositions. The reason is that such propositions cannot be converted *simply*. By simple conversion, it will be remembered, is meant the interchange of subject and predicate without any further change in the proposition. The simple converse of "All S is P," would then be, "All P is S." But the second proposition is not a valid inference from the first, as appears when both are reduced to the hypothetical form. "All S is P," becomes, "If x is S, then x is P," and "All P is S," becomes, "If x is P, then x is S." If, now, these hypothetical propositions were equivalent, it would be possible to construct a valid syllogism with a minor premise which affirms the consequent, since in that case a premise which agreed with the consequent of the one proposition would agree with the antecedent of the other. While then "false obversion" is analogous to the denial of the antecedent, "false conversion"—as the simple conversion of a universal affirmative proposition may be called—is analogous to the affirmation of the consequent. The universal affirmative may, however, be converted by "limitation." This means that the converse must be a particular proposition. The *true* converse of "All S is P," is then, "*Some* P is S." If the original proposition is, "All Virginians are Americans," the valid converse is, "Some Americans are Virginians."

That the particular negative proposition cannot be converted may be inferred from our inability to convert the universal affirmative proposition by the simple interchange of subject and predicate. Remember that the universal affirmative and the particular negative propositions,—the

A and O propositions—are contradictories; and contradictories can neither be true nor false at the same time. If then, A be true, its contradictory, O, must be false. If, now, it were possible to convert O simply, then its converse, O_1, would also be false; since the converse expresses the same meaning as the convertend. But if O_1, is false, then A_1, *its* contradictory, and the converse of A, would have to be true. If then it were possible to convert the O proposition, it would also be possible to pass from the truth of the A proposition to that of its simple converse. But this, as we have seen, is impossible. Therefore it is impossible to convert the O proposition.

Proof that E and I May Be Converted Simply.—That E and I may be converted simply is evident from the meaning of these propositions. It may also be demonstrated by reference to the hypothetical proposition and the square of opposition. Let the E proposition be "No S is P." Its simple converse is then "No P is S." Let us now reduce each of these to the hypothetical form. The original proposition is equivalent to,

(1) If x is S, it is not P; or
(2) If x is P, it is not S.

And its converse is equivalent to,

(1) If x is P, it is not S; or
(2) If x is S, it is not P.

But it is evident that the two pairs of hypothetical propositions are identical. Consequently E and its simple converse are identical in meaning.

E may then be converted simply. What of I? Suppose I to be true. Then its contradictory, E, is false. But if E is false its converse E_1, is also false. And if E_1 is false *its* contradictory, I_1, is true. Again, if I be false, E is true,

E_1, is true, and I_1, is false. Since then from either the truth or the falsity of I we can validly infer the truth or the falsity, as the case may be, of I_1, the conversion of I, like that of E, is a valid process.

Contraposition of the E Proposition by Limitation.—In the case of the I proposition, contraposition is manifestly impossible; for the obverse of an I proposition is an O proposition, and an O proposition can not be converted. Also, on account of the impossibility of converting (simply) the universal affirmative proposition, it is impossible to obtain a simple contrapositive of the universal negative, or E proposition. For the obverse of E is A; and A, as we have just said, cannot be converted simply. It is possible, however, to obtain the *obverted converse* of the E proposition without coming down to the particular level. And if we are content with a particular proposition as the result of the process, E may be obverted and then converted by limitation,—a process which may be called "contraposition by limitation."

Immediate Inference in the Case of Individual Propositions.—Singular, or individual propositions, as we have seen,[1] are treated as universal. This gives rise to no difficulty in obverting, in converting, or in contraposing. Thus, "Socrates was an Athenian," is a proposition of type A. Its obverse is, "Socrates was not a non-Athenian"; its converse (by limitation), "Some Athenian was Socrates"; its partial contrapositive, "No non-Athenian was Socrates"; and its full contrapositive, "Every non-Athenian was a non-Socrates." Similarly, "Oliver Cromwell was not a king," is a proposition of type E. Its obverse is, "Cromwell was a non-king"; its converse, "No king was Cromwell"; its contrapositives (by limitation), "Some non-king was a Cromwell," and, "Some non-king was not a non-Cromwell."

[1] See p. 21.

Certain complications arise, however, when we attempt to apply the rules of *opposition* to individual propositions. In the first place, since the subject of each individual proposition denotes a class of but one member,[1] individual propositions have no subalterns. There is, accordingly, in their case, no "square of opposition." And it is impossible to distinguish between the contrary and the contradictory. "Socrates was an Athenian," and "Socrates was not an Athenian," are contraries; but each is also the contradictory of the other.

Summary.—*Ways of Saying the Same.*—In contrast with the contrary and the contradictory, which express meanings opposed to that of the original proposition, the obverse, the converse, and the contrapositives should be thought of as ways of expressing the same (or at least a part of the same) meaning as is expressed by the original proposition. For example, "All S is P," "No S is non-P," "No non-P is S," and "All non-P is non-S," express the same judgment. In like manner, "No S is P," "No P is S," "All S is non-P," and "All P is non-S," express the same judgment. To sum up the ways of saying the same which we have discussed:—

All categorical propositions may be obverted.

Simple conversion is valid for E and I; simple contraposition for A and O.

A may be converted, and E contraposed by limitation.

Suggestions for Practice.—In our subsequent work it will be of considerable importance to be able (a) to express any categorical proposition in strict logical form; (b) to change it from affirmative to negative or from negative to affirmative; (c) to change the order of its terms; and (d) to express it in hypothetical form (if it is a universal proposition). Operation (b) is, of course, obversion; while (c) can be accomplished by conversion or contraposition. If

[1] See p. 21.

one has occasion to interchange the subject and the predicate of a universal affirmative proposition, this may be done by limitation. But conversion by limitation has the drawback that it is irreversible. One comes down, so to speak, from the universal to the particular level, and is unable to rise again to the higher level. For this reason it is usually better to employ contraposition in the case of the A proposition. Contraposition may then be thought of as a substitute for conversion, that is to say, as an operation which is to be employed in those cases in which simple conversion is impossible. Another way of putting this is to say that propositions which cannot be converted *simply* may be *converted by contraposition.* Thus E and I may be converted simply, while A and O may be converted by contraposition.

Exercises.—1. Which of the following propositions is the obverse of (a)? The converse? The partial contrapositive? The full contrapositive? The contrary? The contradictory? The subaltern?

(a) All sheep are ruminants.
(b) No non-ruminants are sheep.
(c) No sheep are non-ruminants.
(d) All non-ruminants are non-sheep.
(e) Some sheep are not ruminants.
(f) Some ruminants are sheep.
(g) No sheep are ruminants.
(h) Some sheep are ruminants.

2. By what process or combination of processes may we derive (b) from (a)? (d) from (b)? (h) from (f)? (a) from (c)?

3. What is the relation of (e) to (f)? (e) to (g)? (a) to (e)? (a) to (g)?

4. Suppose (a) were known to be false, what could then be inferred concerning the others? Suppose (a) to be true?

5. Suppose that our knowledge of sheep and ruminants were limited to the information contained in (f), what would we

know as to the truth or falsity of each of the other propositions.

6. If we knew only that (e) is false, what could be inferred?

7. Given the proposition, "No men are infallible," write its converse, its contradictory, its obverse, its contrary, and its subaltern.

8. Discuss the following inferences. Are they valid or invalid, and why?

(a) Some H is W,	therefore	Some W is H.
(b) Some H is not W,	"	Some H is W.
(c) No H is W,	"	Some H is not W.
(d) All H is W,	"	Some H is W.
(e) All H is W,	"	No non-H is W.
(f) All H is W,	"	All W is H.
(g) No H is W,	"	No W is H.
(h) All H is W	"	All non-W is non-H.
(i) Some H is not W,	"	Some W is not H.
(j) Some H is not W,	"	Some non-W is H.
(k) No H is W,	"	No non-W is non-H.
(l) No H is W,	"	All non-H is W.
(m) No H is W,	"	All H is non-W.

9. Write an original categorical proposition of type A; then write its obverse, its contrary, its contradictory, and its two contrapositives.

10. Write an original categorical proposition of type E; then write its obverse, its contrary, its contradictory, and its converse.

11. Are these inferences valid or invalid? Explain.

(a) Some Americans are not white, therefore some white men are not Americans.

(b) No horses are wild, therefore all horses are easily controlled.

(c) Some people are rich, therefore some people are not poor.

(d) All liars are contemptible, therefore no liar is worthy of respect.

(e) Generosity is a virtue, therefore selfishness is a vice.

(f) Only the virtuous are truly noble, therefore no ignoble person is truly virtuous.

(g) All wise men are good, therefore all good men are wise.

12. State the relation between

(a) Good men are wise.

(b) Unwise men are not good.
(c) Some unwise men are good.
(d) No good men are unwise. (Creighton.)

13. Name the logical process by which we pass from each of the following propositions to the succeeding one:

(a) All metals are elements.
(b) No metals are non-elements.
(c) No non-elements are metals.
(d) All non-elements are non-metals.
(e) All metals are elements.
(f) Some elements are metals.
(g) Some metals are elements. (Jevons.)

14. Of the following propositions, which can be inferred from (a)? From which can (a) be inferred? Which contradict (a)? Which do not contradict (a), but cannot be inferred from it?

(a) All just acts are expedient acts.
(b) No expedient acts are unjust.
(c) No just acts are inexpedient.
(d) All inexpedient acts are unjust.
(e) Some unjust acts are inexpedient.
(f) No expedient acts are just.
(g) Some inexpedient acts are unjust.
(h) All expedient acts are just.
(i) No inexpedient acts are just.
(j) All unjust acts are inexpedient.
(k) Some inexpedient acts are just acts.
(l) Some expedient acts are just.
(m) Some just acts are expedient.
(n) Some unjust acts are expedient. (Jevons.)

15. Deal with these propositions as follows:—(1) Express each in strict logical form as a categorical proposition; (2) obvert each; (3) convert (or contrapose) each; (4) express each as a hypothetical proposition.

(a) Only prepaid messages will be delivered.
(b) Pious men only are fit to be ministers of religion.
(c) Only animals are sentient beings.

16. Transform the following propositions in such a way that, without losing any of their force, they may all have the same subject and the same predicate:

(a) No non-P is S.
(b) All P is non-S.
(c) Some P is S.
(d) Some non-P is not non-S. (Jevons.)

17. Describe the logical relations, if any, between each of the following propositions and each of the others:

(a) There are no inorganic substances which do not contain carbon.

(b) All organic substances contain carbon.

(c) Some substances not containing carbon are organic.

(d) Some inorganic substances do not contain carbon. (Keynes.)

18. Write the obverse, the contradictory, and the converse (or contrapositive) of each statement contained in the subjoined paragraphs. (Notice that in the case of *individual* propositions the contradictory is the same as the contrary. Connectives, such as "but" and "however," as well as parenthetical expressions and hypothetical sentences, may be disregarded.)

SECTIONALISM AND THE NATION

No phenomenon of our recent political history is more familiar than the steady increase of national authority at the expense of the state governments. Officials and bureaus at Washington already direct many affairs that were formerly under local regulation, and the tendency to transfer power from the states to the nation does not seem to diminish. We are told that modern means of communication, by drawing the ends of the continent together, are effacing the often artificial boundaries of the states.

But if the states are losing power and individual character, the great sections into which geography and climate divide the country show no signs of political degeneration. In a recent article, Professor Turner of Harvard University, one of our most clear-sighted historians, points out that improved communication has not put an end to sectional differences in the United States any more than it has in Europe—where it has actually had the effect of stimulating national rivalries. Fortunately, we have not the ancient conflicts of race, religion, and language that Europe has to complicate our living, but the close student of our history will find sectionalism a constant and sometimes the determining element in our politics.

It has been so almost from the first. The strong resistance of New England to the foreign policy of Jefferson and to the War of 1812, the victory of the young West over the older East with Jackson, the nullification movement in the South in 1832, the long

struggle between the sections over the extension of slavery that ended in the Civil War, the free-silver movement, the repeated struggles over the issue of high protection or low protection or no protection all started in the opposition of one or another section to policies that other sections were strong enough to carry into effect—to the injury, as it appeared, of the protesting region.

When the Federal Reserve system was established it had to be organized frankly on a regional and not on a centralized plan. The so-called "agricultural bloc" in Congress is not partisan, but sectional, in composition. It is criticized mainly by the people of the Eastern and Northeastern sections and defended by those of the Western and Southern sections.

Professor Turner thinks that the inevitable sectionalism of so immense a country shows itself in other ways. He thinks the conflict over prohibition and its enforcement is sectional; that our literature is not a single thing, but a "choral song of many sections"; that there is sectionalism in our culture, as appears in clearly marked differences in manners and in point of view.

That we recognize the existence of sectionalism does not mean that we condemn it. As we have said, it is inevitable in a country so wide as ours. It saves us from too monotonous a uniformity and too overwhelming a sweep of national emotion. Our national life is broader, richer and more interesting because of the differences among New Englanders, Virginians, Texans, Middle-Westerners and Californians. Whether the states will continue to decay and the nation become in the end formally as well as actually a federation of sections instead of states, no one can tell; but it is certain that we shall always have sectionalism with us, and it is the part of the statesman, though he may recognize and make use of the forces that lie in sectional feeling, to help his own neighbors and constituents to a broad instead of a narrow conception of their interests as a part of the nation.

(*Youth's Companion,* Jan. 18, 1923.)

19. Deal in the same way with paragraphs from any book, magazine, or newspaper. (Notice, however, that exclamatory, interrogative, and imperative sentences are not logical propositions.)

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CHAPTER V

THE CATEGORICAL SYLLOGISM

The Search for Valid Forms.—We have seen that validity is a matter of the form rather than of the subject-matter of an argument. Indeed much of our discussion thus far might properly be called *a search for valid forms.* For example, we have found that the forms, "All A is B, therefore All B is A," and "Some A is not B, therefore Some B is not A," are untrustworthy. On the other hand, the following are formally correct:

1. All A is B, therefore Some B is A.
2. All A is B, therefore No A is non-B.
3. No A is B, therefore No B is A.
4. No A is B, therefore All A is non-B.
5. Some A is B, therefore Some B is A.

These, as the reader will recall, are cases of obversion and conversion. And many other valid forms or "moods" of "immediate inference" might be adduced. In our study of the hypothetical syllogism we have found two moods which are valid,—the *modus ponens,* in which we pass from the affirmation of the antecedent to the affirmation of the consequent and the *modus tollens,* in which we pass from the negation of the consequent to the negation of the antecedent.

Valid Moods of the Categorical Syllogism.—In our study of the categorical syllogism our purpose remains the

same,—to discover forms which can always be relied on. A little later we shall be in a position to demonstrate the validity of certain moods, some of which are the following:

(1)	(2)	(3)	(4)
All A is B	No A is B	All A is B	No A is B
All C is A	All C is A	Some C is A	Some C is A
∴ All C is B	∴ No C is B	∴ Some C is B	∴ Some C is not B

In these arguments, no matter what values we assign to A, B, and C, if the premises are true, the conclusion inevitably follows. Indicating the quantity and quality of every proposition by one of the letters, A, E, I, or O, these four valid moods are represented respectively by the combinations AAA, EAE, AII and EIO. In the same way, "All A is B; All C is A; therefore No C is B," is an example of the mood AAE; while, "No A is B; No C is A; therefore No C is B," is an example of the mood EEE. These last, however, are obviously invalid.

The Figures of the Syllogism.—The number of the combinations which must be considered in our search for valid forms is greatly increased by the fact that there are four *figures* of the categorical syllogism. The figure of the syllogism depends upon the position of the "middle term," that is to say, of the term which is found in both the premises. It is clear that there are four possible arrangements. The common or middle term may be the subject of the first premise and the predicate of the second, as in the examples given above, all of which are syllogisms of the *first* figure; or the middle term may be the predicate of *both* premises,—an arrangement which is called the *second* figure; or it may be the *subject* of both, when the syllogism is of the *third* figure; or, finally, the arrangement may be the exact reverse of that found in the first figure, that is to say, the middle term may be the predicate of the major and the

subject of the minor premise, and this is called the *fourth* figure. The four figures may be symbolized thus:

I	II	III	IV
M — P	P — M	M — P	P — M
S — M	S — M	M — S	M — S
S — P	S — P	S — P	S — P

In these syllogistic skeletons, M represents the middle term, while S and P stand respectively for the minor term and the major term. This symbolism is traditional. The use of M for the middle term is self-explanatory. S represents the minor term and P the major term, for the reason that the former is always the subject, and the latter the predicate of the conclusion. The major premise is so called, because it contains the major term and the minor premise in like manner receives its name because it contains the minor term. Before deciding upon the figure of a syllogism, one should be sure that the premises are written in the proper order. The major premise, i.e., the one which contains the predicate of the conclusion, should be written first. Therefore when your attention is called to a syllogism, the first step is to find the conclusion. Then observe which is the major and which the minor term. If the statement containing the major term does not come first, transpose the premises.

The Equivalence of the Figures.—One must not suppose that the four figures are irreducible and absolutely distinct. On the contrary it is possible to reduce any mood of any figure to an equivalent syllogism of any other figure which may be preferred. For it is obvious that by the conversion (or partial contraposition) of the major premise the first figure may be changed to the second, and *vice versa*. And by applying the same procedure to the minor premise the third figure may be reduced to the first, and the fourth

to the second. This process has been called *reduction.* Historically the chief interest has lain in the reduction of the other figures to the first, which was considered to be the perfect figure. For our purposes, however, it will be sufficient to reduce any syllogism of the third or fourth figures to the first *or* the second. Since these are interchangeable it is of course easy to complete the reduction when it is desirable to do so.

Exercises in Reduction.—What is the mood and figure of each of the following? Reduce all to moods of the first or the second figure.

1. All P is M; No S is M; therefore No S is P.
2. All S is M; All M is P; therefore All S is P. (Transpose the premises.)
3. All A is B; No C is B; therefore No C is A.
4. All M is N; No R is N; therefore No M is R.
5. No R is S; All T is S; therefore No T is R.
6. All X is Y; All W is X; therefore All W is Y.
7. No P is M; Some M is S; therefore Some S is not P.
8. All M is P; Some M is S; therefore Some S is P.
9. No M is P; All M is S; therefore Some S is not P.
10. All P is M; No M is S; therefore No S is P.
11. All men are mortal, and all men are fallible; therefore some fallible beings are mortal.
12. No birds are quadrupeds; Some quadrupeds are domesticated animals; therefore Some domesticated animals are not birds.

Reduction to the Hypothetical Form.—We have just seen that all syllogisms which are not already of the first or second figures may easily be reduced to one or the other of these figures. If now we can reduce syllogisms of the first and second figures to the hypothetical form, we shall be in possession of a method by which the validity of any categorical syllogism may be tested. Consider these moods of the first figure:

All M is P	All M is P	All M is P	All M is P
All S is M	No S is M	Some S is M	Some S is not M
∴ All S is P	∴ No S is P	∴ Some S is P	∴ Some S is not P

All that is required to transform these into hypothetical syllogisms is to throw their common major premise into the hypothetical form. "All M is P" is equivalent to, "If x is M, then x is P." Let x = *any S*. We now have,

If any S is M, it is P.

All S is M	No S is M	Some S is M	Some S is not M
∴ All S is P	∴ No S is P	∴ Some S is P	∴ Some S is not P

We now have four hypothetical syllogisms with the same major premise. Which two of them are valid?
Next consider the moods of the first figure which have a major premise of type E.

No M is P	No M is P	No M is P	No M is P
All S is M	No S is M	Some S is M	Some S is not M
∴ No S is P	∴ No S is P	∴ Some S is not P.	∴ Some S is not P

"No M is P," is equivalent to, "If x is M, it is not P." If x = *any S*, we have,

If any S is M, it is not P.

All S is M	No S is M	Some S is M	Some S is not M
∴ No S is P	∴ No S is P	∴ Some S is not P.	∴ Some S is not P

Here again we have four hypothetical syllogisms with the same major premise. Which are valid?

The valid moods of the first figure are seen to be equivalent in each case to a hypothetical syllogism in which the minor premise affirms the antecedent. In other words, the first figure of the categorical syllogism reduces to the *modus ponens*. We shall now show that the second figure reduces

to the *modus tollens*. We shall prove this for but one mood, leaving it to the reader to prove it for the rest.

No P is M =	If any S is P, it is not M.
All S is M	All S is M.
∴ No S is P	∴ No S is P.

We notice that the minor premise disagrees with the consequent of the major. The syllogism is therefore valid,—a modus tollens.

Exercises.—After reducing the syllogisms on pages 2 and 3 to the first or the second figure, reduce them to the hypothetical form. Are they valid or invalid? If valid, is each a *modus ponens* or a *modus tollens?* If invalid, what is the fallacy?

The Major Premise Always Universal.—The reader will have observed that we have not examined any syllogism whose major premise is particular. For if the major premise of a syllogism in the first or second figure were particular, it would be impossible to reduce the syllogism to the hypothetical form. Suppose the major premise to be, "Some A is B." Thrown into the hypothetical form this would not give us, "If x is A, then x is B"; for this is the equivalent of a *universal* proposition. The most that we could validly infer would be that, "If x is A, then x *may* be B," or, "If x is A, there is a certain degree of probability that it is also B." But it is evident that from a proposition such as this, no trustworthy inference can be made. Accordingly, if the major premise is particular and the minor universal the premises must be transposed. If there is no universal premise, the syllogism is of course invalid.

On the other hand, if there are two universal premises, and with one of them as major the syllogism appears to be invalid when thrown into the hypothetical form, the premises should be transposed; for there are a few cases in

which the syllogism will be found to be valid when one of the universal propositions is chosen as major premise, altho apparently invalid if the other is taken.

How to Test Categorical Syllogisms.—Let us sum up our results thus far. We have learned that it is always possible to reduce syllogisms of the third and fourth figures to equivalent syllogisms of either the first or the second figure. We have also discovered that syllogisms of the first and second figures may be reduced to the hypothetical form, in which, if they are valid, they appear respectively as the *modus ponens* or the *modus tollens*. We are therefore in possession of a procedure by means of which any categorical syllogism may be shown to be either valid or invalid. It will be helpful to recapitulate the various steps which are included in the general method. In each case as many steps are to be employed as may be necessary.

1. If the major premise is particular, transpose the premises.

2. If the syllogism is of the third or the fourth figure, reduce it to the first or the second, as may be the more convenient. In some cases this can be accomplished most readily by converting (or contraposing) the minor premise; in others, by transposing the premises.

3. Reduce the syllogism thus obtained to the hypothetical form.

4. Test it by applying the rule of the hypothetical syllogism.

5. If the syllogism has two universal premises, and the test indicates invalidity, transpose the premises, and repeat the test.

When the premises are transposed, the major term becomes the minor, and the minor term becomes the major. If, after the transposition of the premises, the syllogism appears to be valid, we must observe whether the conclusion

is the same as the conclusion suggested, or, if it is not, whether the conclusion which was originally suggested can be derived by valid obversion, conversion, or contraposition from the conclusion which has been shown to follow from the premises. And, in general, in testing a syllogism our aim is to determine whether the given conclusion may be validly inferred from the given premises. In order to do this we first discover what conclusion does follow from the premises, and then inquire whether the given conclusion is a valid inference from it. For example, suppose we have the premises, "Some A is B," and "No A is C." Do these support the conclusion, "Some C is not B?" Transposing the premises, and reducing to the first figure, and then to the hypothetical form, our syllogism becomes,

If any B is A, it is not C.
Some B is A.
Therefore Some B is not C.

Inasmuch, however, as the simple conversion of the O proposition is not valid, "Some C is not B," cannot be inferred from, "Some B is not C." Consequently the suggested conclusion cannot be derived from the given premises.

Exercises.—1. What conclusion follows from each of these pairs of premises?

(a) All M is N, and No M is P?
(b) Some M is not N, and All N is P?
(c) Some M is N, and All N is P?
(d) Some M is N, and No M is P?

2. Test the validity of the following arguments:

(a) No M is N, and No M is P; therefore No N is P.
(b) Some M is N, and No P is M; therefore Some P is not N.
(c) Some M is N, and No P is N; therefore Some M is not P.
(d) All N is M, and Some P is not M; therefore Some P is not N.

Certain Moods which are Obviously Invalid.—There are several types of argument appearing in the guise of a categorical syllogism which are obviously invalid, and which may therefore be rejected without going to the trouble to reduce them to the hypothetical form.

1. *In every valid categorical syllogism there are three terms, and only three;* for otherwise there would be no term common to both premises, and in the equivalent hypothetical syllogism, the minor premise, instead of affirming the antecedent or denying the consequent of the major, would be entirely irrelevant, i.e., would say nothing about the major premise at all.

2. *If both premises are affirmative, the conclusion must be affirmative;* for it is evident that the corresponding hypothetical syllogism, if valid, must be a *modus ponens.*

3. *If both premises are negative, the syllogism is invalid.* For the major premise will then be equivalent to, "If x is A, then x is not B"; and as the minor premise is also negative, it can neither agree with the antecedent nor disagree with the consequent.

4. *If either premise is negative, the conclusion must be negative.* Suppose it is the major premise which is negative. Then if the equivalent hypothetical syllogism is valid, the minor premise must either agree with the antecedent of the major, in which case the conclusion must agree with the consequent; or the minor premise must disagree with the consequent, in which case the conclusion must disagree with the antecedent. But in either case the conclusion must be negative. On the other hand, if it is the minor premise which is negative, the syllogism must be a *modus tollens,* and the conclusion must be negative.

5. *If both premises are particular, the syllogism is invalid.* We have already shown that in the first and second figures the major premise must be universal; and there is no way

by which a syllogism with two particular premises can be reduced to an equivalent syllogism having a universal premise.

6. *If either premise is particular, the conclusion must be particular;* for when the syllogism is reduced to the first or the second figure, the particular premise will be the minor premise, and its subject will be the minor term, and consequently, also, the subject of the conclusion.

The breach of any of these requirements produces a corresponding fallacy. We may accordingly list the following more obvious fallacies of the categorical syllogism, for which the student should be on the lookout before applying the more laborious test:

1. The fallacy of *four terms.*
2. The fallacy of *a negative conclusion from two affirmative premises.*
3. The fallacy of *two negative premises.*
4. The fallacy of *an affirmative conclusion from a negative premise.*
5. The fallacy of *two particular premises.*
6. The fallacy of *a universal conclusion from a particular premise.*

Exercises.—Are the following syllogisms valid or invalid? Give reasons for your answers.

1. No A is B, and No B is C; therefore No C is A.
2. All A is B, and all C is D; therefore all A is D.
3. All A is B, and no B is C; therefore some C is A.
4. Some A is B, and All C is A; therefore all C is B.
5. Some farmers are Americans, and some Americans are Pennsylvanians; therefore some farmers are Pennsylvanians.
6. No squares are circles, and no triangles are circles; therefore no squares are triangles.
7. All men are fallible; Mr. Jones is a man; therefore Mr. Jones is fallible.

8. No men are infallible; Mr. Jones is a man; therefore Mr. Jones is not infallible.

9. No Pennsylvanians are Virginians; Mr. X is a Virginian; therefore he is not a Pennsylvanian. (The major premise is equivalent to, "If x is a Pennsylvanian, he is not a Virginian.")

10. Some negroes are Americans, and all negroes are men; therefore some men are Americans. (Transpose the premises, and then convert the minor premise and the conclusion.)

11. No priests are saints, but some priests are martyrs; therefore some martyrs are not saints. (Convert the minor premise.)

12. All good citizens vote at elections; all good citizens pay their taxes; therefore some who pay their taxes vote at elections. (How can this syllogism be reduced to the first figure?)

13. All patrons of the arts and sciences are public benefactors. All public benefactors are worthy of honor. Therefore some persons who are worthy of honor are public benefactors. (Reduce to the first or second figure.)

14. No college men are cruel; all cowards are cruel men; therefore no college men are cowards.

15. All horses are quadrupeds, and all quadrupeds are vertebrates; therefore all horses are vertebrates.

16. Some Americans are Texans, and some Texans are residents of Galveston; therefore some Americans are residents of Galveston.

17. All Virginians are Americans, and some Americans are Negroes; therefore some Negroes are Virginians.

18. No elements are compounds; coal is not a metal; therefore coal is a compound substance.

19. If the conclusion is an A proposition, of what quantity and quality are the premises?

20. What combinations of premises may yield as conclusion a proposition of type E? I? O?

Approval of Syllogisms by Inspection.—Not only are there, as we have seen, certain moods which are obviously invalid, but there are also certain moods in each figure which one will soon learn to recognize as valid, even without reducing them to the *modus ponens* or the *modus tollens*. And, in general, if there is a universal premise of which

the middle term is the subject, and if the other premise is *affirmative,* the syllogism is, or may readily be reduced to, a *valid* syllogism of the first figure. If, however, after the syllogism has been reduced to the first figure, the minor premise is *negative,* the syllogism is invalid. These principles have been repeatedly exemplified. And it is clear that an affirmative minor premise in the first figure will *affirm the antecedent,* while a negative minor premise will *deny* the antecedent, when the major premise is expressed in hypothetical form.

Likewise, if there is a universal premise of which the middle term is the predicate, and if the premises *differ* in *quality,* the syllogism is, or may readily be reduced to, a valid syllogism of the second figure. If, however, after the syllogism has been reduced to the second figure, the premises are of the *same* quality, the syllogism is invalid; for if the premises differ in quality, it is clear that when the major premise is expressed in hypothetical form the minor premise will *deny the consequent;* while if the premises are of the same quality it will *affirm* the consequent.

Summary: Rules for Testing Categorical Syllogisms. —It is always possible to determine the validity or invalidity of a categorical syllogism by reducing it to the hypothetical form. It is often easier, however, to apply certain rules (which may be justified by an appeal to the rules of the hypothetical syllogism). These rules of the categorical syllogism may be classified under three heads: those which hold for all categorical syllogisms; those which hold for syllogisms of the first figure; and those which hold for syllogisms of the second figure.

A. Rules for Syllogisms of any Figure. (See the section on "Obvious Fallacies," The six rules need not be repeated here.)

B. Special Rules for Syllogisms of the First Figure.

1. The major premise must be universal.
2. The minor premise must be affirmative.

C. Special Rules for Syllogisms of the Second Figure.

1. The major premise must be universal.
2. The premises must differ in quality.

Testing Syllogisms by Circles.—The validity or invalidity of categorical syllogisms may often be made clear to the eye by the help of diagrams. These diagrams, however, should be considered as illustrations rather than as proofs. This method of representation depends on the convention that the terms of a proposition denote classes, which may stand to one another in various relations of inclusion and exclusion. In other words, these classes may or may not overlap. Now it is obvious that as regards inclusion and exclusion two given classes may stand in any one of five different relations. (a) They may be co-extensive; in other

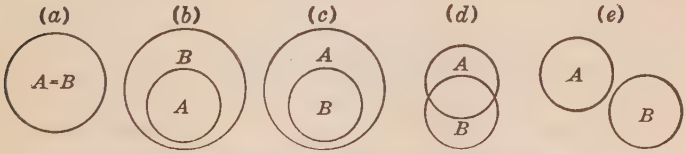

Fig. 2.—The Five Relations of Two Classes.

words, each of them may completely include the other.
(b) The second may include the first, and more besides.
(c) The first may include the second, and more besides.
(d) Each may partly include and partly exclude the other.
(e) Each may completely exclude the other.

When we attempt to represent logical propositions by means of these diagrams, we find that most of our typical forms are ambiguous. Thus, "All A is B," may mean either (a) or (b). "Some A is B," may mean (a), (b), (c), or (d). On the other hand, "No A is B," has only one meaning, (e). But, "Some A is not B," may mean either (c), (d), or (e). In order to be "on the safe side," however, it is customary to assume that in the case of the A proposition the two classes stand in relation (b); and that in the case of the I and the O proposition they stand in the relation (d).

Suppose, then, that we wish to test the syllogism, "All A is B, and No B is C; therefore No C is A." It may be represented thus:

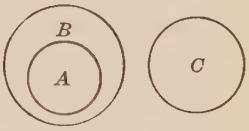

Fig. 3.—A Valid Syllogism.

It is clear, in this case, that any diagram which correctly represents the premises will represent the conclusion also. Consequently the syllogism is valid. Consider, however, the syllogism, "All A is B, and All C is B; therefore All C is A." In this case the circles which represent the premises may be combined in more than one way. Two of these ways are shown in the diagram; and both of these contradict the conclusion. Consequently this syllogism is invalid.

Fig. 4.—An Invalid Syllogism.

In general, then, if it is impossible to draw circles to represent the premises without at the same time representing the conclusion, the syllogism is valid. While if there are two or more constructions each of which represents the premises, and at least one of which is at variance with the conclusion, the syllogism is invalid.

Exercises.—1. How many moods are valid in the first figure? In each of the other figures? (Consider all possible combinations of A, E, I, and O, taken three at a time; as AAA, AAE, AAI, etc.)

2. Reduce each of the valid moods of the third figure to the first. Of the fourth to the second.

3. Show that each of these complies with the general rules of the syllogism, and also with the special rules of its figure.

4. Show furthermore that each may be reduced to a valid mood of the hypothetical syllogism.

5. Draw diagrams of all the valid moods of each of the four figures.

6. Prove that if one premise of a syllogism is negative the conclusion must be negative. You may quote the rule for the hypothetical syllogism. (Be sure that your proof is *general,* i.e., that it holds not simply for one or two syllogisms but for *any* syllogism.)

7. In the same way prove the following:

(a) In the first and third figures, the minor premise must be affirmative.

(b) In the second figure, the premises must differ in quality.

(c) In the first and third figures, if the conclusion is negative, the major premise must be negative.

(d) In the second figure, if the conclusion is affirmative, the syllogism is invalid.

(e) In the third figure, if the conclusion is universal, the syllogism is invalid.

(f) In the fourth figure, if the conclusion is universal and affirmative, the syllogism is invalid.

8. Prove, in general, that the subject of a universal conclusion must be either the subject of a universal premise or the predicate of a negative premise.

9. Prove, in general, that the predicate of a negative conclusion must be either the subject of a universal premise or the predicate of a negative premise.

10. *Definition.*—A term is *distributed* if it is the subject of a universal proposition or the predicate of a negative proposition. Prove,

(a) If a term is distributed in the conclusion, it must be distributed in its premise.

(b) The middle term must be distributed in at least one of the premises.

11. Complete the following enthymemes, and demonstrate their validity:

(a) He is the owner of a large store, and must therefore be a wealthy man.

(b) None but material bodies gravitate; therefore air is a material body.

(c) Socrates is mortal because he is a man.

(d) Agricultural education will eventually increase the output of farm products; therefore agricultural education should be promoted.

(e) Because this triangle is equilateral, its angles are equal.

(f) Prohibition ought to be enforced, for it is a part of the constitution.

(g) Since a horse is a quadruped it is a vertebrate.

(h) This man is sure to succeed because he is industrious.

(i) This is a good book; therefore its author was a good man.

(j) Only material bodies gravitate; ether does not gravitate.

(k) Fire is a benefit to industry because it creates work.

(l) There should be no restriction of debate in the United States Senate; for complete freedom of debate is necessary to prevent ill-advised legislation.

(m) This man is educated, and therefore does not want to work with his hands.

(n) Logic and mathematics both furnish excellent formal discipline; therefore the latter may be regarded as a branch of the former. (Creighton.)

(o) Luxury should be encouraged in those who can afford it, because it makes a market for the products of industry.

(p) Animals only are sentient beings; therefore all plants are insentient.

(q) Business enterprises are most successful when managed by those who have a direct interest in them; therefore enterprises carried on by the state are not likely to succeed. (Creighton.)

(r) This man is a Protestant; for he exercises the right of private judgment.

(s) He could not face bullets on the field of battle, and is therefore a coward.

(t) A miracle is incredible because it contradicts the laws of nature.

(u) There can be no such thing as an omniscient mind, since all thinking is a succession of mental states.

(v) The student of history is compelled to admit the law of progress, for he finds that society has never stood still.

12. Are the following valid or invalid? Explain.

(a) This building has a cross-crowned spire; churches have cross-crowned spires; hence this is a church. (W. T. Harris.)

(b) No natural tree is cross-crowned; this object is cross-crowned; therefore it is not a natural tree. (Ibid.)

(c) Some church steeples have belfries; this is a church steeple; hence it has a belfry. (Ibid.)

(d) All men are bipeds, and all men are rational; hence some rational beings are bipeds.

(e) No vegetable has a heart; a good lettuce has a heart; therefore a good lettuce is not a vegetable.

(f) Some workingmen are Tories, and some workingmen are tailors; therefore some tailors are Tories. (Joseph.)

(g) No clergyman may sit in Parliament, and some clergymen are electors to Parliament. Therefore some electors to Parliament may not sit in it. (Ibid.)

(h) All nitrogenous foods are fleshforming. All grains are nitrogenous. Therefore all grains are fleshforming.

13. Criticize this gem from Mark Twain: "Jim said bees wouldn't sting idiots; but I didn't believe that, because I had tried them lots of times myself, and they wouldn't sting me." —Huckleberry Finn, Chap. VIII.

14. Test these syllogisms:

(a) To make a corner in wheat produces great misery; to make a corner in wheat is gambling; therefore all gambling produces great misery.

(b) Some rocks are sedimentary; granites are not sedimentary; therefore some rocks are not granites.

(c) No insects have eight legs; spiders have eight legs; therefore spiders are not insects.

(d) All negroes have curly hair; some natives of Africa do not have curly hair; therefore some natives of Africa are not negroes.

(e) Those who can find out things for themselves are but little dependent on education; men of genius can find things out for themselves; therefore men of genius are but little dependent upon education.

(f) No fragrant flowers are scarlet; some geraniums are scarlet; therefore some geraniums are not fragrant.

(g) All true roses bloom in summer; a Christmas rose does not bloom in summer; therefore it is not a true rose.

(h) Many soldiers were wounded in the great battle. My friend was a soldier. Therefore he was wounded in that battle.

(i) All wine-producing countries are warm countries. Spain is a warm country. Therefore Spain produces wines.

(j) All Parisians are Frenchmen. Some artists are Frenchmen. Therefore some artists are Parisians.

(k) No mammals are indigenous to Australia. All mammals are vertebrate. Therefore some vertebrates are not indigenous to Australia. (L. J. Russell.)

(l) No English railways are badly managed; no badly managed railways are profitable. Therefore no English railways are unprofitable. (Ibid.)

(m) No conservative government is revolutionary; all revolutionary governments are unsuccessful; therefore all conservative governments are successful.

(n) All good lessons develop initiative; but lessons in which children are told everything are not good lessons; therefore they do not develop initiative. (Ibid.)

(o) The atmosphere cannot be a carrier of electricity; for metals are conductors of electricity, and the atmosphere is of course not a metal. (Hibben.)

(p) Whatever is opposed to our industrial prosperity is to be regarded as a serious evil. All European wars are serious evils. Therefore they are opposed to our industrial prosperity. (Ibid.)

(q) The courageous are confident, and the experienced are confident. Therefore the experienced are courageous. (Ibid.)

15. Point out the fallacies in the following arguments:

(a) All members of the finance committee are members of the executive committee. No members of the library committee are members of the finance committee; therefore no members of the library committee are members of the executive committee. (Hibben.)

(b) I am sure that he must have known of the plan; for only members of the committee knew of it, and he was a member of the committee. (Ibid.)

(c) Only those messages which are prepaid will be delivered. This message has been prepaid; and therefore it will be delivered. (Ibid.)

(d) That writer must be a follower of Plato; for all followers of Plato are idealists, and he is an idealist. (Ibid.)

(e) Everyone desires happiness; virtue is happiness: therefore everyone desires virtue. (Ibid.)

(f) The principles of justice are variable; the appointments of nature are invariable; therefore the principles of justice are no appointments of nature. (Ibid.)

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CHAPTER VI

AMBIGUITY AND DEFINITION

Examples of Ambiguity.—

1. Light is given off by the sun; feathers are light; therefore feathers are given off by the sun.
2. A receiver of stolen goods should be punished; anyone who purchases goods that were stolen is a receiver of stolen goods, and should therefore be punished.
3. All the interior angles of a triangle are equal to two right angles; this angle is an interior angle of a triangle; therefore this angle is equal to two right angles.
4. No cat has nine tails; every cat has one tail more than no cat; therefore every cat has ten tails.
5. No news is good news; therefore all news is bad news.

A school boy, casually looking through a textbook in logic, remarked, "This is a joke-book." He had happened upon exercises such as these. These arguments are all manifestly correct, if we consider only their form. Yet they are absurd. In each case one of the terms is ambiguous. Thus in No. 5, it is clear that the introductory word is not a sign of a universal negative proposition. It is really a modifier of the subject, and the proposition has the force of an A proposition. No. 4, in like manner, is vitiated by the ambiguity of the phrase "No cat." In one place "No" introduces a universal negative proposition; in the other place, "no" is a modifier of "cat." No. 3 confuses two meanings of "All." *All* may mean *every,* or it may mean *the sum-total of.* In formal logic it is regularly employed

in the former sense, but in the major premise of No. 3 it is evidently employed in the latter. In No. 1, there is, of course, a play on the word "light." In No. 2, the ambiguity may not at first sight be so evident; but in the major premise the phrase, "Receiver of stolen goods," is employed in a technical sense, while in the minor premise it denotes anyone into whose possession the goods may chance to come.

The Vagueness of Common Words.—In a perfect language every sign would always represent one and the same idea. Punsters might perhaps object, but serious men would rejoice if such a situation could be attained. For it is unnecessary to say that our language, measured by this test, is far from perfect. Scarcely an English word but has several meanings in good usage, unless perchance certain technical terms which have been recently coined for particular purposes. What, for example, is the meaning of the word *church?* Is a "church" a building of wood or brick or stone, or is it the body of worshippers who are wont to assemble in the building, or is it the larger organization of which such a congregation is a part? What, again, is a *secretary*—a young woman, a piece of furniture, or a bird? Thus we might go on *ad libitum* multiplying examples of the diverse meanings of words. A *house* is a human habitation; it is also a dynasty, as the House of Hapsburg, or a legislative body, as the House of Commons. *A burn* is a sore produced by heat; and also a brook. A *glass* is a tumbler, a mirror, or a small telescope. A *train* is a line of connected cars on a railroad; and it is, or rather used to be, "the portion of a gown which was dragged behind the wearer."

How Words Change Their Meanings.—A word which has two or more meanings is said to be *vague* or *equivocal.* In some cases it is probable that the equivocality has resulted from an accidental similarity in the sound of words which

were originally different. More frequently one meaning has arisen from another because of an underlying resemblance, or by an association of ideas. Thus the various meanings of *train, glass, house,* etc. are obviously connected. In some cases an older "etymological" meaning persists alongside a more modern signification. Thus *to prevent* has the etymological meaning, "to come before," as well as the modern meaning, "to hinder." Similarly the word *aggravate,* which more properly means "to increase," "to make heavier," has come to mean, "to vex," or "to exasperate." The word *prove,* from the Latin *probare,* meant merely "to probe," or "to test," as, for example, in the saying, "the exception which proves the rule." Now it has come to mean, "to establish by a process of reasoning." The word *doctor* meant originally a teacher; then one who had a degree from a university; then one licensed to practice medicine.

The Role of the Context.—The vagueness or equivocality of words is not dangerous unless it is forgotten. Almost any word, as we have seen, has several meanings. As life has become more complex and men's ideas more numerous, instead of coining a new word for every new thing or thought, we have usually compelled old words to do double duty. So that, if almost any word, say *sensation,* or *constitution,* or *democracy,* or any of the words discussed in the preceding paragraphs, were written on the blackboard by itself, no one but the writer could be sure what sense he had in mind. As soon, however, as a word is used in connected discourse, some of the meanings suggested by it when it stood alone are seen to be no longer possible. For example, if the word *sensation* appears in the sentence, "The Professor made clear to his class in psychology that there is a difference between sensation and perception," the word cannot mean "exciting and surprising information."

While if the same word appears in the statement, "It is the policy of a certain newspaper to feature some sensation in each issue, and if it cannot find a sensation it creates one," the word clearly does not have the meaning which it has in psychology. The sentence or paragraph in which a word is found, and which thus restricts its meaning is called the *context*. When the meaning of a word is unclear, it is usually possible to tell from the context what sense is intended. In spoken language we are even more dependent upon the context than in written or printed language. For example *to, too,* and *two,* sound exactly alike; yet we usually have no difficulty with them, and many children and semi-illiterate adults do not feel the need of the distinction in spelling. To take a somewhat more extreme case, French *oui* and English *we,* French *eau* and English, *Oh,* are nearly the same in sound; yet a person familiar with both languages never has any trouble on this score. The most extreme case of the function of the context is afforded by sentences containing missing words. Thus the sentence, "John ——— James, and James rubbed the place and began to cry," is fairly explicit, although one of the words has been omitted.

A Definition as a Special Sort of Context.—In a later chapter after a discussion of classification, we shall consider the traditional account of definition.[1] We shall there find that *definition equals genus plus differentia.* It is sufficient for the present to say that this means that a term is adequately defined when we have given the class to which it belongs, (the genus) and also the respects in which it differs from other members of the same class (the differentia). For our present purpose a less formal account of definition will be sufficient. And what has been said of the role of the context will be a good point of departure. A definition, then,

[1] See Chap. XI.

may be considered as a somewhat more formal and self-conscious employment of a context to make clear the meaning of a term.

Our Reasons for Defining.—It may be helpful to distinguish two different purposes for which a definition may be sought. In the first place one may wish to increase his working vocabulary. The word in question may be entirely unfamiliar. One looks it up in a dictionary, and finds that it means the same as some other word with which he is already familiar. Thus one might look up the word *ere* and find that it means the same as *before*. Now the latter may indicate relative position either in space or in time. It is accordingly a vague term. Yet if a child learns that *ere* means the same as *before, ere* becomes a part of his vocabulary. Or a boy in reading a tale of adventure may come across the word *canoe*. He hunts it up in the dictionary, or asks someone what it means, and learns that a canoe is a boat. Here again no precise meaning has been assigned to the word in question, for there are many different kinds of boats. Yet the boy will thus make the word his own, and be able to use it. Perhaps, however, the words are not "looked up." Perhaps no one is asked what they mean. In course of time they will probably be learned from the context. In this way, indeed, we learn the meaning of most of the words which we employ.

In the second place, one may wish to attain greater precision in the use of terms with which he is already familiar. This is the sort of definition with which the logician is especially interested. If he would avoid the fallacy of "four terms," he must give each term one meaning and adhere to this meaning throughout the entire argument. In this sense definition is like the mathematician's assigning of values to *a, b, c,* etc. Any value may be given to his constants, but a value having once been given, this must

be the value of the letter wherever it appears in the calculation.

Definition by the Use of Synonyms.—Definition in both senses may be accomplished with the help of synonyms. A *synonym* is a word which means the same, or almost the same, as another word. Thus *begin* and *commence* are synonyms. Usually, however, there is more divergence in meaning between synonyms than there is between this pair. For even words which are etymologically equivalent have in most cases picked up different sets of associates in the course of their history. Thus *mansion* and *dwelling* should mean the same. But the one came into the English language from the Latin by way of the Norman French, while the other is Saxon. And as the Normans in the centuries succeeding the conquest had large habitations while the Saxons dwelt in huts, our customary distinction arose. A writer's *style* depends chiefly upon his ability to find the synonym which has the proper associations and which exactly conveys the meaning which he has in mind.

In the preceding section, in our discussion of the meaning of *ere* and *canoe* we sufficiently illustrated the way in which synonyms may be used to introduce us to the meaning of unfamiliar words. It may at first sight not be clear that the enumeration of several synonyms may restrict the meaning of a term to one precise sense, as is required for the purposes of the exact thinker; for the synonyms of a word are usually no less vague than the given word itself. The very fact, however, that a word may have several synonyms no one of which covers exactly the same area of meaning,—a fact which at first seems so baffling,—enables a master of style to make his meaning absolutely clear. Each of the synonyms, I have said, has a certain *area of meaning,* but these areas are not quite coincident. Each synonym may then be thought of as giving a *locus* within which the meaning in-

tended must be found. These *loci* overlap, and the precise meaning intended must be found in the small area common to all of them; just as if I should be told that a certain man lives less than three blocks from the railway station, within half a mile of the post office, in a house the number of which is 867. The directions would not be quite so satisfactory as they would be if I were told, at once, that he lives at 867 Third Avenue; but I should probably be able to puzzle out his precise *locus*. Similarly if an *ascent* is said to be *high,* to be *steep,* to be *abrupt,* to be *precipitous,* I get the meaning more accurately from this context than I could get it from any one of these adjectives by itself. Again, if a certain man is said to be a *minister,* a *diplomat,* an *agent,* and a *representative,* although each of these terms is vague, the man's status is pretty well defined.

The Limits of Definition.—When our purpose in defining is to acquaint someone else with the meaning of what is for him a "new" word, we are naturally expected to define it in terms which are already familiar to him. Thus Dr. Johnson's famous definition of *net* is a horrid example of how not to do it, because his definition contains the long and unfamiliar word *reticulated.* We expect unfamiliar words to be defined by the help of those which are familiar; we expect complex ideas to be explained by reference to ideas which are simpler. And we frequently reproach the dictionary makers, because we say, the definitions are often harder to understand than the words defined. Moreover, dictionary definitions are often circular. That is to say, when we look for *A,* the dictionary tells us that it means *B,* and when we look for *B,* we find that *B* in turn means *A*. But we should not blame the dictionary makers too severely, for their task is a difficult one. In the task of defining we must either move in a circle, or we must stop somewhere and explicitly recognize that certain terms are

indefinable or *not-further-analyzable*. For it is manifest that we cannot continue indefinitely from the unfamiliar to the more familiar and from this to the still more familiar, and so on; neither can we continue indefinitely from the relatively complex to the simple and from this to the still simpler. We shall come after a while to words or ideas than which none simpler or more familiar can be found. Thus if we are challenged to tell what is meant by *blue* or *pleasant,* it is not likely that we shall succeed in finding a definition any simpler or more familiar than the idea *blue,* or pleasant, itself. If terms such as these are to be defined at all, it can only be done, as Bertrand Russell has said, by the "method of pointing." That is to say the most that we can do is to put a man, if such there be, who has never had an elementary experience like *blue* or *pleasant* in a situation where he may get it for himself; and if he lacks the appropriate organs for the apprehension of the experience in question,—if, for example, he is color blind,—there is no way in which we can transfer it from our mind to his.

The Enumeration of Meanings.—In order to be perfectly sure that the area of meaning covered by a given term is exactly the same each time the term appears in a given argument or exposition, it is advisable to enumerate the various senses in which the term is employed. For example, if we employ the term *law,* we should be perfectly clear in our own minds whether we mean *natural* law, *moral* law, or *civil* law. And if we are quite sure that we mean *civil* law, it may still be well to inquire whether we are thinking of *statute* law, i.e. law which has arisen through definite legislative enactment, or of *common* law, by which is meant law which has arisen from use and wont as interpreted by a long sequence of judicial decisions. Thus Goldwin Smith, through failure to analyze the sense in which he was em-

ploying the term *law,* argued fallaciously that the existence of a power (above nature) is implied by the phrase "laws of nature," constantly employed by science; for "wherever there is a law there must be a lawgiver, and the lawgiver must be presumed capable of suspending the operation of law."[1] Whatever may be our theological position, we can scarcely expect the existence of God and the possibility of miracles to be established by an argument so short and simple as this. Indeed it is only of statute law that we are justified in saying that "law implies a law-giver." *A law of nature,* as the phrase is used in science, is merely a description of the sequence in which certain events take place.

The Use of Antonyms.—In attempting to disentangle the various meanings of a word, we may be hindered as well as helped by the dictionary. Not all the meanings given in the dictionary are really different. Before referring to the dictionary it is well to take an inventory, so to speak, of the various senses in which we ourselves have been accustomed to use a given word. In doing this it is helpful to think not only of the synonyms but also of the antonyms of the word. Just as synonyms are words which mean the same, or nearly the same, so *antonyms* are words which have opposed meanings. For example, if we wish to distinguish two meanings of *light,* we might speak of one of them as the opposite of *dark* and the other as the opposite of *heavy.* In the same way we might distinguish *natural* as opposed to *artificial* from natural as opposed to *supernatural; real* as opposed to *non-existent* from real as opposed to *worthless; good* as opposed to *wicked* from good as opposed to *useless.* When the meanings of a word cannot be distinguished by means of antonyms, two or more sets of synonyms may be employed, or the distinction may be made clear by the use of paraphrases.

[1] Bode, *Outline of Logic,* p. 26.

Exercises.—1. In the case of each of these words, distinguish at least two meanings, by means of antonyms, synonyms, or paraphrases. Be sure that the meanings are really different:—

law	good	block	moral	spectacles	school	spring
nature	church	crack	natural	cradle	course	glass
real	secretary	office	sensible	religion	bureau	train
evil	rubber	capital	pitiful	state	hand	house

2. Illustrate each meaning by using the word in a sentence or paragraph so constructed that in each case the context excludes any meaning except the one to be illustrated.

3. Find as many other words as you can which may be correctly employed in at least two different senses.

The Arbitrariness of Definition.—A story is told of a certain lady who remarked that "the French call an article of daily consumption *'pan,'* and the English call it *'bread,'* but everyone knows that its real name is *'Brod.'*" Except for a few "onomatopoetic" words, i.e. words whose sound imitates their meaning, as, for example, *hiss* or *buzz,* there is no necessary relation between words and their meanings. In other words, there is a certain arbitrariness about the use of language. What a word means is a matter of "convention," not in the sense that words have received their meaning as a result of the agreement of men now living, but in the sense that ages ago men just started to use certain linguistic roots in certain ways and we still use them in much the same way. If, however, all men now living, or even all who use a certain language, should agree upon a reshuffling of words and meanings, the new combinations, after we became accustomed to them would be as good as the old. Therefore it is usually a waste of time to argue about the "right" or "true" meaning of a word. In some cases, to be sure, the student of literature attempts to fix the true meaning of an appeal to etymology. But no one would insist that "to come before" is more truly the meaning

of *prevent* than "to hinder." Usage is the criterion; and good usage frequently departs very far indeed from suggestions of etymology. In logic, at any rate, we are not concerned with disputes of this kind. We assign meanings to our terms much as the mathematician gives values to his letters, *a, b, c,* etc. A thinker has the right to define his terms as he pleases, provided that he then remains faithful to the meanings which he himself has given them. Nevertheless, it is usually wise to depart as little as possible from common usage; for if words are employed in needlessly strange and unfamiliar senses, the reader is likely to be confused, and the writer himself is in danger of slipping back now and then into the customary meaning.

In passing judgment upon any argument or exposition, it is then not enough to assure ourselves of its formal correctness. It is not enough to make sure that no rule of the syllogism is violated. We must also be constantly on the alert for the detection of sliding terms. We must inquire whether the sense to which a word is restricted in one part of the context is also the sense required elsewhere. If a writer is found to employ his terms inconsistently, he has been convicted of ambiguity; and ambiguity is the logician's unpardonable sin.

Vagueness and Ambiguity.—In Chapter I, in speaking of the benefits to be expected from a course in logic, we mentioned the *habit of defining our terms,* as one of the good habits which this study has a tendency to foster. Now far be it from us to suggest that there is any *panacea,* any one prescription which will infallibly cure all logical distempers; but the prescription which comes nearest to being such a cure-all is this, that *we should avoid ambiguity.* We need not be afraid to employ *vague* terms; for almost all terms have several meanings, but the vagueness of language must not be allowed to lead us into ambiguity, which con-

sists essentially in the neglect of important differences in meaning. When a given term is used in two statements it frequently happens that for one of these statements to be true the term must be understood in a certain sense, while for the other to be true the same term must be taken in a quite different sense. In other words, the context is inconsistent, giving now one and now another signification to the same word.

A striking illustration of this use of a sliding or ambiguous term is found in the argument advanced by the pessimist Schopenhauer for the thesis that there must necessarily be more pain than pleasure in the world. Schopenhauer argues (1) that pleasure consists in the satisfaction of desire; (2) that desire is a form of pain. From these premises it seems to follow that pleasure is only a relief from pain, and therefore that pain is the normal and usual experience, since there may be pain without pleasure, but pleasure is impossible except by contrast with pain. This is Schopenhauer's famous doctrine of the "negativity of pleasure." The argument, however, is vitiated by what is in effect the "fallacy of four terms." For the middle term, *desire,* is ambiguous. In the first premise, *desire* must be interpreted broadly, so as to include not only *conscious* wants, but also any *needs* whatsoever, whether conscious or unconscious, the satisfaction of which produces pleasure. While, in the second premise, only *conscious* needs can be meant, inasmuch as pain which is not conscious would not be pain at all in any significant sense of the term. At any rate, an *unconscious* pain could not be allowed to offset a pleasure in our attempt to determine which exists in the greater amount.

Summary and Retrospect.—Let us take a backward glance over this and the preceding chapter. In order that an argument may deserve to be called good its premises

must, of course, be true; it must also be expressed in language which is clear and precise; and it must be in proper form. It is not necessary to employ complete syllogisms in our thinking and writing. Enthymemes are usually more effective. Nevertheless a training in syllogistic theory is of great value. For one thing, it teaches us to supply the hidden premises, which are often more important than those which are expressed; for the hidden premises contain the assumptions which—unexpressed and unacknowledged—underlie the argument; and it is only after these assumptions have been made explicit that we are in a position to inquire concerning their truth or falsity. Moreover, if we express the argument in syllogistic form, we are likely to discover such cases of ambiguity as may exist; for if one of the three terms is ambiguous, there are virtually four terms instead of three.

Exercises.—1. Show that each of the following arguments is vitiated by a fallacy which is virtually that of "four terms." Explain the exact shift of meaning which has taken place in each:

(a) No evil should be allowed that good may come of it; all punishment is an evil; therefore no punishment should be allowed that good may come of it.

(b) Whatever is dictated by Nature is allowable; devotedness to the pursuit of pleasure in youth, and to that of gain in age, are dictated by Nature; therefore they are allowable.

(c) Transitory things are unimportant; the evils of life are transitory, and therefore are unimportant.

(d) Every people has the right to determine the form of government under which it would live; the members of the Communist Party are people; therefore they have a right to determine the form of government under which they would live.

2. The following arguments are taken from the Congressional Record of October 2, 1922. Are they good or bad? So far as possible, express them in syllogistic form. If they are

enthymemes, supply the missing parts. Point out errors in the use of the syllogism; invalid obversion or conversion; ambiguous use of terms; or statements contrary to facts. (In some cases, the same error will fall under more than one of these heads.)

(a) I voted against prohibition . . . because I did not believe it either right or expedient to take the police power from the states, where it had rested for more than 100 years, and place it with the nation.—Hon. Walter M. Chandler of New York.

(b) I voted against . . . prohibition because I did not feel that it was right that outlying, sparsely settled States should determine the drinking habits of the people of my State (New York).—*Ibid.*

(c) I voted against prohibition because the overwhelming sentiment of the people of my district was against it.—*Ibid.*

(d) It is urged that the plea for light wines and beers is but the opening of an argument for the restoration of the saloon and bad liquor. Only designing men will make this contention, and only the ignorant and unsophisticated will be influenced by it. The eighteenth amendment prohibits "the manufacture, sale, or transportation of intoxicating liquors," and the Supreme Court of the United States stands guardian over the rights of the people in the matter of its enforcement. Until the eighteenth amendment is revoked there can be no manufacture, sale, or transportation of intoxicating liquors, and Congress is powerless to pass any laws violating this amendment as long as the Supreme Court functions and does its duty.—*Ibid.*

(e) This definition (of intoxicating liquor as that which contains more than one-half of one percent of alcohol) is based upon experience and reason. In order to enforce the law against intoxicating liquor it is necessary to prohibit all alcoholic liquors with enough alcohol in them to produce intoxication when the largest amount of the liquid is consumed within the time when the alcohol could accumulate in the human system. If the opponents of this standard could point out some State of the Union where they have enforced prohibition honestly and effectively with a 2 or 3 percent beer exemption, there would be some basis for their appeal.—Hon. Edward C. Little of Kansas.

(f) We should always be grateful to the strong and brave

Americans who first pointed out the dangers of the League of Nations and halted our permanent entrance into foreign entanglements. If we had gone into the League of Nations, we would be asked to settle trouble, in the making of which we had no part, and which resulted from foreign jealousies and intrigues.—Hon. William F. Kapp of Iowa.

(g) If the bill (the Esch-Cummins railroad act) cannot be repealed, I am in favor of very substantial amendments to the act, because it violates many fundamental principles of proper legislation.—Hon. W. Frank James of Michigan.

(h) Instead of this law creating a guaranty it terminated the one made by President Wilson in December, 1917. The contention that this six-month guaranty was based on "watered stock" is therefore on its face a pure fabrication. It was not based on capital stock or on valuation, but was based on the "average of the net operating income for the three-year period ending June 30, 1917," as agreed to by President Wilson.—Hon. Halvor Steenerson of Minnesota.

(i) People of all classes want the Government to accept the Henry Ford proposal for Muscle Shoals because they believe that Henry Ford is a practical business man and manufacturer who has the money, the managerial brain, and the inclination to successfully operate a plant of this magnitude and to really make it a great development; and because they know that Henry Ford will share prosperity and spread prosperity over the whole community by paying employees of all classifications better and fairer wages than other interests would pay them; and also because they believe he will sell the products of the project . . . at a fairer price to the consumer than some other interest would.—Hon. John Philip Hill of Maryland.

3. Go to some current periodical and select a half dozen arguments, and discuss them as you have the above excerpts from the Congressional Record. Remember, however, that pure narration, description, or exposition is not argument, and may therefore be disregarded.

4. Discuss the following arguments.[1] (1) Point out the hidden premises or assumptions. (2) Tell which premises, if any, are contrary to fact. (3) If there are ambiguous terms explain the shift of meaning in the case of each. (4) If the

[1] These directions apply, not only to No. 4, but to the subsequent exercises as well.

argument is good, express it in regular logical form. (In some cases the conclusion must be supplied.)

(a) For a good many years before the war, aliens from southern and eastern Europe largely outnumbered those from northern and western Europe. Under the new law these numbers are nearly equalized, so that if all nationalities fill their alloted quotas, the so-called "newer" immigration can not contribute more than about one-half of our annual inflow. This fact is biologically of great significance. In the fiscal year ending June 30, 1922, deducting emigrants from immigrants, we gained in Nordic stock, and lost in the natives of southern and eastern Europe.

(b) Those who attribute solely to the present percentage restriction the need of labor in certain industries are either wholly ignorant of the facts, or are intentionally trying to mislead the public in the effort to break down all restrictions and to flood the country with cheap labor. In this connection it should be realized that (1) there has been a very considerable emigration of alien labor during the recent period of business depression and unemployment; (2) if all countries filled up their quotas, which they have been doing during the past year, there would be an annual inflow of over 350,000; . . .

(c) If we want the American race to continue to be predominantly Anglo-Saxon-Germanic, of the same stock as that which originally settled the United States, wrote our Constitution, and established our democratic institutions; if we want our future immigration to be chiefly made up of kindred peoples from northern and western Europe, easily assimilated, literate, of a high grade of intelligence, able to understand, appreciate and intelligently support our form of government, then the simplest way to accomplish this purpose is to base the percentage limitation upon an earlier census than that of 1910, i.e., before southern and eastern Europe had become the controlling element in our immigration. (*The Scientific Monthly,* Dec. 1922, pp. 563ff.)

5. (a) Innate differences are known only when opportunities for practice have been equalized. This is to say that a test or examination *per se* can never be regarded as a test of native capacity.

(b) That the scores in these examinations are materially

affected by social experience is attested by certain other lines of evidence. An appreciable rise in them was found to accompany length of residence in this country on the part of the foreign-born, amounting to two years of mental age when the first five-year interval is compared with the fourth. There seems little reason to suppose that the first comers were more able than their successors.

(c) Among the men from England only 8.7 per cent were rated D or less, while among the Poles the percentage making these low ratings was almost 70. In general the Scandinavian and English-speaking countries stand high in the list, while the Slavic and Latin countries stand low. The disparity here is so great as to suggest other than selective factors at work. The great differences are shown between languages most removed in kinship, which coincide with the most marked differences in the extension of popular education and popular culture.

(d) College seniors who were given four forms of Alpha at intervals of three weeks showed striking improvement in the second and third trials. The practice effect for the lower half of the group was much greater than for the higher, indicating that the lower scores were in greater degree affected by the novelty of the tasks. (Sci. Monthly, Feb., 1923, *The Army Intelligence Findings,* pp. 185ff.)

6. (a) If free will were true, science would be impossible, since the success of an experiment would be no warrant for its success upon repetition. We find chance nowhere in our dealings with nature. Neither with dead or living matter, in the field of the telescope or in that of the microscope, has anyone yet found an exception to the causal law. Chance is driven to lurk in the realm of psychology because it has been driven out of every other branch of natural science successively.

(b) Every act involves energy. A "free" act would mean that this energy came from nowhere and did not previously exist in any form, since, if it did, the existing energy would be the efficient cause of the act. But perhaps, since you deny the law of causality, you will have no scruples as regards the law of the conservation of energy.

(c) The interactionist has positive arguments too. The facts of our especially intense consciousness at moments of decision; of the assumption by one part of the brain of the function of

another part which has been injured or even lost, and the close connection between pain and danger to the body all point to an influence of mind on body.

(d) If the body can act with perfect efficiency as an automaton, why does it not always do so? If the mind cannot by its presence or nature modify conduct and so aid survival, how does it happen that the body has a mind? More important yet is the question why consciousness takes such a form that it seems to be essential to the proper control of the body to insure its survival. (Slosson, P. W., *Fated or Free,* pp. 8, 9, 19, 20.)

7. In 1912 a police lieutenant in New York hired three gamblers to put a certain man out of the way. The gamblers, in turn, hired four "gunmen," who did the actual murder. The gamblers were granted immunity if they would testify against the lieutenant and the gunmen, which they did. The lieutenant and the gunmen were convicted and electrocuted.

(a) It was right to grant immunity to the gamblers, for without their testimony no one could have been convicted.

(b) The guilt of the lieutenant was greater than that of the three gamblers, because it was he who first suggested the murder.

(c) The guilt of the four gunmen was greater than that of the gamblers because they did the actual, physical murder.

(d) Capital punishment is justified, because it protects society. (See Cox, George Clarke, *The Public Conscience,* pp. 93ff.)

8. (a) Suicide should be forbidden by law, because the life of each individual belongs to the state.

(b) Insurance policies ought to be valid in cases of death by suicide. The great majority of such cases are due to a disordered mind. (*Ibid.,* p. 131.)

9. It was rumored that General Gouraud would challenge Sir Walter Scott to fight a duel on account of certain statements in the latter's "Life of Napoleon." Scott was entirely willing to settle the matter *off* the field of honor. He would show the general his authorities, but if he should ask "any apology or explanation for having made use of his name" it was "his purpose to decline it and stand the consequences." He was aware he could "march off upon the privileges of

literature" but he had no taste for that species of retreat; "if a gentleman says to me, I have injured him, however captious the quarrel may be, I certainly do not think as a man of honor, I can avoid giving satisfaction without doing intolerable injury to my own feelings, and give rise to the most malignant animadversions." (*Ibid.*, p. 144, quoted from *Century Magazine,* Vol. 59:478.)

10. A bankrupt left some plate in his wife's possession. She induced a friend to take it to a pawnshop, where he pawned it in his own name, gave his own note to repay the money, and immediately upon receipt of the money went back to the bankrupt's wife and delivered the money to her. (*Ibid.*, p. 208.)

(a) The man who pawned the silver was blameless, because he acted as a friend and derived no profit from the transaction.

(b) The creditors had no right to the silver, because the bankrupt had given it to his wife.

11. In 1917 the Supreme Court upheld the "blue sky" laws, which provided for the licensing of dealers in stocks, bonds, and other securities. Justice McKenna rendered the decision.

(a) The integrity of securities "can only be assured by the probity of the dealers in them and the information they are required to give. This assurance the states deem necessary for their welfare to require, and that requirement is not unreasonable or inappropriate."

(b) "We cannot stay the hands of government upon consideration of the impolicy of its legislation. Every new regulation of business meets challenge. But the policy of a state and its expression in laws must vary with circumstances."

(c) "The statutes burden business, it is true, but burden it only that under its forms dishonest business may not be done. Expense may thereby be caused, and inconvenience, but to arrest the power of the state by such considerations would make it impotent to discharge its functions. It costs something to be governed."

12. In the *Cyclopedia of American Government,* Article "Suffrage," Vol. III, p. 446, it is said, "If the penalties for disfranchisement decreed by the Fourteenth Amendment were enforced, it would mean the loss of at least three representatives in Congress to such a state as Louisiana or Mississippi. But Congress is not likely to take upon itself the enforcement

of the penalty, for the ratification of those amendments was procured only by counting the votes of states which acted under duress, and the requirement of such ratification as a prerequisite to readmission is considered to have been of doubtful constitutionality. Moreover, serious doubt has been growing as to the justice and the expediency of the suffrage conditions which were forced upon the southern states. The foremost leaders among the negroes themselves have avowed their approval of both property and educational tests, if fairly administered, since each of them would serve as a spur to greater efforts on the part of the negroes in thrift and education." (*Ibid.*, p. 456.)

13. Property is in no situation absolute. The state, as well as its various subdivisions, taxes—i.e. seizes property without property compensation. Franchises, rights of way, right to do what one will with one's own (admission to hotels and theaters) rights to charge what one will for one's services (rate regulation of railroads, etc.), contracts and many similar property rights are held, not precariously indeed, but very definitely under the will of the state; and in Eminent Domain this principle finds its fullest and most overt expression. While constitutional countries habitually do recompense those whose property is seized, it often occurs that no compensation in money or goods can adequately compensate. One does not want one's ancestral home destroyed even if the condemnation price is more than the home is worth commercially; but this makes no difference. Absolute property, according to statutes and the common law, very evidently inheres in the state alone. (*Ibid.*, p. 474.)[1]

14. "According to Mr. Bryan's premises," said Doctor Stiles, "all germs that cause disease must have been created in the beginning as they exist today. If it is to be conceded that these germs were created originally in some form other than as disease germs the theory of evolution stands admitted.

"Obviously, as Adam was the last animal created and as the animals were not created until after the plants, it is unthinkable that any of the numerous germs that cause disease were created

[1] The reader will of course understand that Mr. Cox's book is not a work on logic. It is a very admirable attempt to introduce the case method into the teaching of ethics. The citations which I have made have been considerably modified, in some cases, in order to make logical exercises of them. This is especially true of Nos. 7, 8, and 10.

after Adam. As disease germs are dependent for their existence on animals and plants in which they cause disease, it is clear that these germs could not have been created or existed prior to the creation of their victims."

A challenge of this deduction would be an admission that the germs were not created as they are today, but that they later evolved into disease germs, but that would be an admission of evolution." *Philadelphia North American,* April 1, 1923.

15. Let these believers in the "tree man" come down out of the trees and meet the issue. Let them defend the teaching of agnosticism or atheism if they dare. If they deny that the natural tendency of Darwinism is to lead many to a denial of God, let them frankly point out the portions of the Bible which they regard as consistent with Darwinism, or evolution applied to man. They weaken faith in God, discourage prayer, raise doubt as to a future life, reduce Christ to the stature of a man, and make the Bible a "scrap of paper." As religion is the only basis of morals, it is time for Christians to protect religion from its most insidious enemy. (William J. Bryan, the *New York Times,* 1922.)

16. An objection commonly raised [to the eudaemonistic or pleasure-pain theory in ethics] is that pleasures and pains of various sorts are incommensurable; that therefore no calculation of relative advantage is possible; and that the eudaemonistic criterion for action is thereby made impracticable and useless.

To this we may reply that the estimation of the relative worth of different kinds of experience is, indeed, often very difficult. But on any theory the decision as to the right is equally complicated and puzzling. The fact that the criterion is difficult to use is no evidence that it is not the right criterion. Which set of consequences will be of most intrinsic worth, it is sometimes impossible to know. But one set *is,* nevertheless, of more intrinsic worth, and the act that secures them is the best act, even though we do not recognize it as such. There will continue to be many differences of judgment as to which of alternative possible experiences is the more desirable. But that uncertainty does not alter the fundamental fact that some experiences *are* intrinsically more desirable than others and

more deserving of pursuit. (Drake, Durant, *Problems of Conduct,* p. 139.)

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CHAPTER VII

SYMBOLIC LOGIC

All logic is symbolic, in the sense that it makes use of symbols, such as S, P, and M, for the terms of the syllogism. In what is known technically as "Symbolic Logic," however, a more radical procedure is attempted. As far as possible, Symbolic Logic avoids the use of ordinary language, and employs symbols, not only as substitutes for the *terms* employed in ordinary discourse, but also as signs of the *relations* which hold between terms or propositions.

The Part-Whole Relation, or Inclusion.—The proposition, "All horses are quadrupeds," may be interpreted as the assertion that the class *horse* is completely included within the class *quadruped*. In other words, the class *horse* is a part of the class *quadruped*. Now a part is usually less than the whole of which it is a part. Hence it has been found convenient to represent the copula of propositions such as "All horses are quadrupeds" by the symbol $<$, which is used in mathematics for "less than." Accordingly, in Symbolic Logic, $<$ is the symbol for *inclusion*. It may be read, "is included within," or "is a part of." While this is the derivation of the symbol for inclusion, we must not, however, insist upon the meaning "less than." For in Symbolic Logic, every class is said to be included within itself, to be *a part of itself;* and, of course, nothing can be less than itself. Thus, using h to represent *horse,* and q to represent *quadruped,* we may write $h < q$. But we may also write $h < h$ and $q < q$. In accordance with our conventions, these

expressions mean, respectively, "The class *horse* is included within the class *quadruped*"; "the class *horse* is included within the class *horse*"; and "the class *quadruped* is included within the class *quadruped*."

The convention that a class may be a part of itself, is likely to puzzle the beginner. How can a thing be a part of itself? The answer is that in Symbolic Logic the word *part* is not used in the ordinary sense. In ordinary language, *part* means "only a part," a portion which is less than the whole. We must accustom ourselves to the use of the word in the sense of "a part at least," a portion which is less than *or equal to* the whole. (In this connection it will be helpful to recall what was said in Chapter IV about the logical meaning of the word *Some*.)

A part which is *less than* its whole is said in logic to be a "proper part." (Compare the expression "proper fraction" in arithmetic.) The relation $a < b$ may then be said to obtain, whether a is a proper part of b or whether a is coextensive with b. In either case we may say that a is included within b. If a and b represent the same group of objects, that is to say if a is identical with b the expression $a < b$ may be replaced by $a < a$. And, in general, if a stands for a class of things, it is always permissible to write $a < a$. In the same way we may always write, $b < b$, $c < c$, $x < x$, etc. These expressions are to be read, "a is included within a," "b is included within b," "c is included within c," etc.

Reciprocal Inclusion, or Identity.—The class *horse* is included within the class *quadruped*, but the class *quadruped* is not included within the class *horse*. This matter-of-fact statement may help us to understand the distinction which has just been made between a mere inclusion and an identity. If h represents *horse* and q represents *quadruped*, we may write $h < q$; but, if the expression is to be true, we cannot write $q < h$. In other words, these expressions are not

equivalent. And, in general, $a < b$ and $b < a$ are not equivalent expressions. Neither of the two can be inferred from the other. There are some cases, however, in which both of these expressions are true. These correspond to the cases in which a universal affirmative proposition may be converted simply. For example, let a stand for "equilateral triangle" and b for "equiangular triangle." We may, in this case, write both $a < b$ and $b < a$. In this case a and b are said to be *identical*. In other words, reciprocal inclusion is equivalent to identity. The symbol employed for identity is the mathematical sign of equality. Thus *equilateral triangle* $=$ *equiangular triangle*. And, in general, if $a < b$ and $b < a$, it follows that $a = b$, and *vice versa*.

Logical Sum and Logical Product.—Two other arithmetical symbols are employed in Symbolic Logic. These are $+$ and $\times$. These symbols, however, like the symbol $<$, have a meaning somewhat different from that which is given to them in arithmetic. To put the matter briefly, a "logical sum," $a + b$, is an *alternative;* a "logical product," $a \times b$, or simply ab, is an *overlapping*. Whatever you may have learned in arithmetic, remember that in Symbolic Logic "plus" means "or," while "times" means "and." Thus *horse* $+$ *mule* means that which is *either* a horse *or* a mule; while *gentleman* $\times$ *farmer*, or simply gentleman-farmer, means one who is *both* a gentleman *and* a farmer.

Sums and Products of Areas.—This usage may seem a little less paradoxical, if we think of the entities to be added or multiplied as *areas*, and their sums and products as *loci*. Let a be the area of an orchard, and b the area of an adjoining barnyard. Then $a + b$ is the sum of orchard and barnyard. If, now, a colt has been given the run of the barnyard, and the gate between the barnyard and the orchard happens to be left open, the colt's range, or *locus*, becomes barnyard *plus* orchard. Hence, if the fence is secure, we know that

the colt will be found somewhere in the barnyard *or* in the orchard. Thus barnyard *plus* orchard is equivalent to barnyard *or* orchard.

In much the same way we may illustrate the idea of a "logical product." A boy, detected in some misdemeanor, promises the judge of the juvenile court that for the space of thirty days he will not leave his native county. After returning to his home he promises his mother that during the same time he will not leave his father's farm. The farm lies partly in the boy's native county and partly in an adjoining county. Now if the boy fulfils both promises, what will be the limit of his wanderings during the thirty days? Manifestly his range will be limited to the "logical product" of the farm and the county; that is to say to the region in which farm and county overlap. If a represents the county and b the farm, the common region is $a \times b$, or simply ab. Observe that in Symbolic Logic, as in algebra, while the $+$ must be written, the $\times$ may be omitted. In other words, when no symbol of operation is given it is understood that the operation intended is multiplication.

Exercises.—1. Imagine a situation in which the boy's *locus* or range would be the logical sum of farm and county.

2. Draw a square and a circle so as to overlap. Enclose their logical product with a red line and their logical sum with a blue line.

3. Let s be a square, and c a circle. Their logical sum is $s + c$, and their logical product is sc. Let the area of s be 16 square inches, and that of c 5 square inches. If, then, the area of $s + c$ is 20 square inches, what is the area of sc? Again, if the area of sc is 2 square inches, what is the area of $s + c$?

4. What is the logical product of a square and a circle which do not overlap?

5. What is the logical sum of two areas one of which is wholly included by the other? The logical product?

6. What is the logical product of the British Empire, the Western Hemisphere, and the North Temperate Zone?

7. What is the logical product of the class *Negro* and the class *American?*

8. The logical sum of *American* and *Caucasian?*

Application to the Relations of Classes.—Let a represent a certain man. Let b represent the class *merchant,* and c the class *farmer.* We are now ready to interpret the following:

$$(1)\ a < b$$
$$(2)\ a < c$$
$$(3)\ a < b + c$$
$$(4)\ a < bc$$

The interpretation of (1) is "This man is a merchant." That of (2), "This man is a farmer." (3) "This man is either a merchant or a farmer." And (4) "This man is both a merchant and a farmer."

The logical sum of the class *merchant* and the class *farmer* is the class which includes all merchants and all farmers. Accordingly every member of this class is either a merchant or a farmer. On the other hand, the logical product of *merchant* and *farmer* is the class which contains only such individuals as are both merchants and farmers.

In general, and more precisely, the logical sum of two classes is the *least* class which contains all the members of the first class and also all the members of the second. The logical product of two classes is the *greatest* class which contains no members but such as are members of both classes.

The Logical Sum Not Disjunctive.—The student will recall that in our study of the alternative proposition, we distinguished between propositions which are really disjunctive and those which are merely alternative. Now the logical sum need not be disjunctive. It corresponds to the alternative rather than to the disjunctive proposition. The alternatives may be mutually exclusive, or they may not. In

other words, the two classes may overlap. For example, in the proposition, "This man is either a merchant or a farmer," there is the possibility that he is both. Or, to recur to the case of two overlapping areas, if we say, "The boy is either in his native county or on his father's farm," he might be in the region common to both. It is manifest, however, that if the product of the two classes or areas is 0, their sum is disjunctive.

Exercises.—If a, b, c, etc. represent classes or areas, interpret the following expressions. (Observe the use of the parentheses. In Symbolic Logic, as in ordinary algebra, when two terms or combinations of terms occur without any intervening sign, it is understood that one is to be multiplied by the other.)

1. $(a < b)\ (b < c)\ (c < d)$
2. $(a + b)\ (c + d)\ (e + f) < gh$
3. $ab + cd + ef + gh < mn + pq$
4. $(ab + cd)\ (ef + gh) < (mn + pq)\ (rs + tu)$

The Meaning of 0 and 1.—What shall we say of the product of two classes which do not overlap? There are two positions which may be taken. Since the product of two classes is the class which is included in both of them, two classes which have nothing in common might be said to have no product at all. In symbolic logic, however, another form of statement is preferred. We say, rather, that when two classes have no common element their product is 0. In other words, we assume that even though the classes do not overlap, there is nevertheless a class which is included within both classes, but that it is *a class without any members*. Such an empty class is known as the "null class." Manifestly, if a class is contained in each of two classes which do not overlap, it is contained in any class whatsoever. We may then define 0, or the null class, as the class which is included within every class. In symbols, x being any class whatsoever, this becomes,

$$0 < x$$

On the other hand, if we should add together all conceivable classes, the sum thus obtained would be the *whole,* or in technical terms, the "universe." The universe, the sum of all classes, is represented in symbolic logic by 1. Inasmuch as the whole includes each of its parts, 1 may be defined as the class in which every class is included. In symbols, x again being any class whatsoever, we have,

$$x < 1$$

The Application of the Symbols to the Relations of Concepts.—The proposition, "All men are mortal," may be interpreted in two different ways: (1) There is a class of men, and there is a class of mortals, and the former class is completely contained within the latter.

(2) There are two concepts, *humanity* and *mortality,* which are so related that the former is an invariable sign of the latter. In either sense, the proposition may be represented symbolically by the expression,

$$a < b$$

If a stands for *men* and b for mortals, the expression is an *inclusion.* It means that the class *men* is included within the class *mortals.* If, on the other hand, a stands for *humanity* and b for *mortality,* the expression is an *implication.* It means that humanity implies mortality. There are then two ways in which we may read the symbol $<$. For example, $a < b$ may be read, "a is included within b," or "a implies b."

Exercises.—Express the following implications by means of symbols:

1. War implies fighting.
2. Loyalty to one's Alma Mater implies an interest in the football team as well as a devotion to studies.

3. Sowing implies reaping.
4. Smoke implies fire.
5. A yellow flame implies the presence of sodium.
6. Strong drink produces babblings and wounds and redness of eyes.
7. Ill health is due to an inherited weakness, to a lack of good food, to a want of pure air, to insufficient exercise, to an invasion of pathogenic bacteria, or to some other physical cause.
8. The extremely hot weather experienced in the month of August is caused by the great length of the days in summer and by the relative perpendicularity of the rays of light.

Primary and Secondary Propositions.—Propositions which assert a relation between concepts or classes are known as *primary* propositions. On the other hand, propositions the terms of which are themselves primary propositions are known as *secondary* propositions. All simple categorical propositions are primary; most hypothetical propositions are secondary. For example, "All horses are quadrupeds," is a primary proposition, because it affirms a relation between the concepts *horse* and *quadruped*. "If this animal is a horse, it is a quadruped," is, however, a secondary proposition, because it affirms a relation between the primary propositions, "This animal is a horse," and "This animal is a quadruped." Expressed in symbols, a primary proposition has but one sign of implication. Thus $a < b$ may mean, "All horses are quadrupeds." The symbolic expression of a secondary proposition, on the other hand, may contain three signs of implication. Thus $(x < a) < (x < b)$ may mean, "If this animal is a horse, it is a quadruped." A secondary proposition may also contain *more* than three signs of implication. For example, $(a < b)\ (c < d) < (e < f)\ (g < h)$ may mean, "If the son is wise and the daughter is beautiful, the father will be proud and the mother will be happy."

We have seen that in primary propositions the symbol $<$

may usually be read either "is included within" or "implies," according as we interpret the proposition as an assertion concerning the relation of classes or of concepts. In secondary propositions, however, $<$ must at least once be read "implies," "it follows that," "therefore," "if . . . , then . . ." It is important to understand that the relation of the antecedent to the consequent of a hypothetical proposition is a relation of implication, as in the examples given in the last paragraph. On the assumption that the hypothetical proposition is true, the truth of the consequent follows necessarily from the truth of the antecedent. The relation between the antecedent and the consequent may therefore be represented by the symbol $<$.

Exercises.—Symbolize the following:

1. If the nation goes to war, its men must fight and its women will weep.
2. If x is a loyal student, he supports the team.
3. This man has sowed the wind; therefore he will reap the whirlwind.
4. The house must be on fire, because the third story is full of smoke and flames appear between the shingles.
5. If the flame becomes yellow, the solution contains sodium.
6. If a man pays his taxes promptly, if he tries to be informed on all public questions, if he votes at all elections, if he refuses bribes of any description whatsoever, if he obeys the laws whether he likes them or not—such a man is a pretty good citizen.
7. In order that a man may be eligible for election to the Presidency he must be a native of the United States, be at least thirty-five years of age, and have resided in this country at least fourteen years.

Equivalent Propositions.—The reader will recall that a reciprocal inclusion may properly be regarded as an identity. That is to say, when a is included within b, and b

is also included within a, it follows that a equals b, and *vice versa*. Expressed symbolically,

$$(a < b)\ (b < a) = (a = b)$$

This may be read, "If a is included within b, and b is included within a, then a is identical with b; and if a and b are identical, then a is included within b, and b within a. Let us now interpret the left hand member of the equation as the product, not of two inclusions, but of two implications. In this sense it may be read, "If a implies b, and b implies a, it follows that a is equal to b; and if a is equal to b, then a implies b and b implies a." The symbol $=$, both as the copula of the principal equation and of the right hand member, represents *equivalence*. The entire equation is a secondary proposition, while the right-hand member is a primary proposition. Remember, then, that two propositions or two concepts are rightly said to be equivalent only in case each of them may be validly inferred from the other. In other words, equivalence is reciprocal implication.

Exercises.—Which of the following relations may properly be represented by $=$, and which should be represented by $<$?

1. This man is a Pennsylvanian; therefore he is an American.
2. If the angle is equilateral, it is also equiangular.
3. Some Americans are Negroes; therefore some Negroes are Americans.
4. All squares are rectangles; therefore some squares are rectangles.
5. No men are immortal; therefore no immortal beings are men.
6. The relation of a proposition of type E or I to its simple converse.
7. That of A to its converse by limitation.
8. That of A to its obverse. To its contrapositive.
9. Of a universal proposition to its subaltern.

Logical Sums and Products of Propositions.—As applied to propositions, the ideas of logical sum and logical product acquire a peculiar significance. The logical product of two propositions is the simultaneous affirmation of both; the logical sum of two propositions is their alternative affirmation. Thus $(a = b)\ (b = c)$ means that both equivalences are true at the same time. $(a < b)\ (b < c)$ means that both implications are true at the same time. $(a = b) + (c = d)$ means that one or the other of the two equivalences is true. And $(a < b) + (c < d)$ means that one or the other of the two implications is true.

Exercises.—1. Read and interpret the following expressions:
(1) $(a < b)\ (b < c) < (a < c)$
(2) $(ab + bd + de + ef) = (m + n)c$
(3) $(a = b)\ (c = d) < (m = n)$

2. Express the following symbolically:
(1) If John struck James, and William threw a stone at Henry, the school will not be given a recess this afternoon.
(2) If the invading army succeeds in crossing the river, the city will be burned and its defenders will be put to the sword.
(3) If the gentleman is at the door, you should ask him to step inside or to take a chair on the porch.

0 and 1 as Propositions.—We have seen that when we deal with classes, 0 is defined as the class which is included within every class; while 1 is defined as the class in which every class is included. These definitions are expressed thus:

$$0 < x$$
$$x < 1$$

In these expressions, x stands for any class whatsoever. If, now, x is understood to mean any proposition, 0 becomes the proposition which implies every proposition, and 1 the proposition which is implied by every proposition. We

might define *0* and *1* in this purely hypothetical fashion without committing ourselves to the belief that there really is a proposition which implies every proposition, or a proposition by which every proposition is implied. Indeed, even if we should suppose the propositions which we have called *0* and *1* to be non-existent, they might still be useful in our calculations. A parallel case is the use of imaginary quantities in mathematics. In mathematics, $\sqrt{-1}$ is not supposed to be an existing quantity, and yet it is used to pass in calculation from real quantities to real quantities. In the same way Symbolic Logic might treat *0* and *1* as imaginary propositions. Even if non-existent, they might help us to pass from real propositions to other real propositions.

It has been found convenient, however, to assign to *0* the meaning "the false," and to *1* the meaning "the true." From this convention, it follows,

(1) that a false proposition implies every proposition, and

(2) that a true proposition is implied by every proposition. At first sight these may seem to be paradoxes or meaningless statements. A plausible interpretation may, however, be given. I would suggest the following. A false proposition implies every proposition in the sense that if what is absurd or self-contradictory is treated as if it were true, we are then required to treat every proposition as true; for no test remains by which any proposition may be condemned as false. It is as if one should say, "if this be true then everything is true; if there are square circles, then there are good bad men and four-footed bipeds." On the other hand, the true proposition is implied by every proposition in the sense that if *any* assertion is to be significant we must assume that truth exists. If nothing is true there is no genuine assertion, and propositions do not exist.

Furthermore, if o implies x, and x implies 1, it follows that o implies 1. Expressed in symbols,

$$(o < x)\ (x < 1) < (o < 1)$$

That is to say, *the false implies the true.* We cannot infer from this, however, $(1 < o)$ or that *the true implies the false.* This, being interpreted, means, it would appear, that it is inconceivable that all propositions should be false (since the proposition that all propositions are false would itself be a proposition); but that it is not inconceivable that all propositions should be true.

Fundamental Principles of the Logical Calculus.—Symbolic Logic has been called the "Algebra of Logic." It essays to resolve reasoning into a species of mathematical calculation. We shall now state several of the fundamental principles of this algebra,—enough of them to give the reader some idea, at least, of the methods of calculation employed. Some of these principles are analogous to the principles of ordinary algebra, while others are quite different. We shall not attempt a rigorous demonstration; but all may readily be proved to hold for areas, if we draw squares or circles to represent a, b, c, etc.

(1) *The Principle of the Syllogism:*—

$$(a < b)\ (b < c) < (a < c)$$

This principle is obvious whether a, b, and c are regarded as classes or as propositions. For if the class a is within the class b and the class b is within the class c, it is clear that the class a is within the class c. On the other hand, if proposition a implies proposition b, and the latter implies proposition c, it is evident that a implies c.

(2) *The Principles of Simplification:*—

$$ab < a;\ a < a + b$$

In ordinary language, these expressions mean that if anything belongs to the class which is common to the class a and the class b, it belongs to the class a; and if anything belongs to the class a, it belongs to the class of things which are found *either* in the class a *or* in the class b. Or, if two propositions, a and b, are both true, then proposition a, considered separately, is true; and if a is true, it follows that a *or* b is true.

(3) *The Laws of Tautology:*— $a = aa$; $a = a + a$. At first sight these laws may seem to be untenable. But the algebra of logic differs from the ordinary algebra in being without either coefficients or exponents. These laws, strange as they at first appear, are in accordance with our definitions of the operations of addition and multiplication. For the class which is common to a and a is manifestly the class a itself; and everything which belongs to the class a may be said to belong *either* to a *or* to a. On the other hand, the simultaneous affirmation of proposition a *and* proposition a is equivalent to the affirmation of a; while the affirmation of proposition a is equivalent to the affirmation of a *or* a. Another way of putting this would be to say that if one is justified in asserting the truth of a, he has a right to repeat the assertion as often as he pleases; and, yet, no matter how often he repeats it, he has said no more than if he had said it but once. While, if one is required to assent to the truth of a, he may be said to have been given his choice between a and, $-a$; and if he is given his choice of a or a, he has no more liberty than if he were required to accept a without the pretence of a choice.

(4) *The Laws of Absorption:*—

$$a + ab = a;\ a(a + b) = a$$

These laws assist greatly in the condensation of expressions. Membership in class a *or* in the class common to class a and

class b is manifestly equivalent to membership in class a itself; and the class common to a and to a *or* b is the class a itself. On the other hand, if a and b represent propositions, the alternative affirmation of a alone *or* of *both* a and b at the same time is clearly equivalent to the affirmation of proposition a without any alternative; while the simultaneous affirmation of a *and* a or b is also equivalent to the affirmation of propositions a by itself.

(5) *The Distributive Law:*—$(a + b)c = ac + bc$. This is so obvious as to need no interpretation. For whether the letters represent classes or propositions, it is clear that to have c and at the same time the choice between a or b is the same as to have the choice between the simultaneous possession of a and c or the simultaneous possession of b and c.

(6) *The Principle of Multiplication:*—

$$(a < b)\ (c < d) < (ac < bd)$$

In finding the product of two inclusions (or of two implications) we may simply multiply the first member of one by the first member of the other, and the second member of the one by the second member of the other. Furthermore, remembering that it is always true that $c < c$, we may write, $(a < b)\ (c < c) < (ac < bc)$. Multiplying, we derive the special formula, $(a < b) < (ac < bc)$.

7 *The Principle of Addition:*—

$$(a < b) + (c < d) < \{(a + c) < (b + d)\}$$

In finding the sum of two inclusions (or implications) we may simply add the corresponding members. Here again, since $c < c$, we derive the special formula,

$$(a < b) < \{(a + c) < (b + c)\}$$

The student will recall the axioms of the ordinary algebra, "if equals be added to equals the sums are equal, and "if equals be multiplied by equals the products are equal." Our last two principles merely substitute $<$ for $=$ in the statement of these axioms.

Exercises.—1. Write a categorical syllogism of the first figure, and mood AAA. Then express it by symbols, and compare your results with the "principle of the syllogism" given above.

2. Condense the following expressions:

(1) $mmmm < nnn$

(2) $m + m + m < n + n$

(3) $m(m + n) = (m + mn)$

3. Expand these expressions:

(1) $(m + n)r$

(2) $(mn + pq)s$

Negation.—Thus far we have made no room for negative terms or negative propositions. Let us now repair this omission. It has been found convenient in Symbolic Logic to denote the negative of a by a', that of b by b', etc. On the principle that a double negative is equivalent to an affirmative, $(a')' = a$. If a is a class, a' includes all of the universe which is not included in a. On the other hand, if a is a proposition, a' is its contradictory proposition. Either for classes or for propositions, then, we have the two expressions,

$$a + a' = 1$$
$$aa' = 0$$

The former means that the logical sum of a class and its negative is the entire universe of discourse; or that, given any proposition and its contradictory, one or the other of them must be true. The latter expression means that the class common to a class and its negative is naught; or that a proposition and its contradictory cannot both be true.

It follows from the definition of o and 1, and from the principle of negation, that the negative of o is 1 and the negative of 1 is o. That is to say, $o' = 1$ and $1' = o$.

If a proposition is expressed as an inclusion (or an implication) it is convenient to represent its negative (contradictory) in a manner analogous to that in which we represent the negative of a term. Thus the negative of $(a < b)$ is $(a < b)'$. Since a proposition and its contradictory cannot both be true, and one of them must be true we have,

$$(a < b)\ (a < b)' = o$$
$$(a < b) + (a < b)' = 1$$

Translating the Ordinary Logic into Symbolic Logic.—All the propositions employed in the ordinary logic may be translated into Symbolic Logic. Let us write some of the propositions as ordinarily written, and the translation in parallel columns.

Ordinary Logic	Symbolic Logic
If A is B, C is D.	$(a < b) < (c < d)$
If A is B, A is C.	$(a < b) < (a < c)$
A is either B or C.	$a < (b + c)$
If A is B, either C is D or E is F.	$(a < b) < \{(c < d) + (e < f)\}$
If A is B, or if C is D, E is F.	$\{(a < b) + (c < d)\} < (e < f)$
All A is B.	$(a < b)$
No A is B.	$(a < b')$
Some A is B.	$(a < b')'$
Some A is not B.	$(a < b)'$

The Symbolic Expression of A, E, I, and O.—A few words of explanation may be necessary concerning negative and particular propositions. If the universal affirmative, or A proposition is interpreted as an inclusion of the subject within the predicate, the corresponding E proposition is an

inclusion of the subject within the *negative* of the predicate. In other words, if "All S is P" means that every member of the class S is also a member of the class P, then "No S is P" means that every member of S is a member of that part of the universe which is outside of P. The particular propositions are readily expressed in symbols if we remember that I is the contradictory of E and O the contradictory of A. Using S and P to represent the subject and predicate, the four typical propositions may then be represented as follows:

$$\begin{array}{ll} \text{A} & (s < p) \\ \text{I} & (s < p')' \end{array} \qquad \begin{array}{ll} \text{E} & (s < p') \\ \text{O} & (s < p)' \end{array}$$

It is important to notice that from the standpoint of Symbolic Logic the propositions ordinarily called particular are to be considered *negative*.[1]

The Processes of Immediate Inference.—It is obvious that all the processes of "immediate inference" can be represented symbolically. (1) As we saw in the preceding paragraph, the relation of *contradiction* is represented by the addition (or the removal) of the prime ′, the sign of negation, to (or from) the entire quantity. The relation of *contrariety* is represented by the addition (or the removal) of the sign of negation to (or from) the predicate term, that is to say the second member of the implication (or inclusion). We have, then, these relations corresponding to those of the "square of opposition":

$$\begin{array}{l} (s < p) \;\; = \left\{ (s < p)' = o \right\} \\ (s < p)' = \left\{ (s < p) \;\; = o \right\} \\ (s < p') \;\; = \left\{ (s < p')' = o \right\} \\ (s < p')' = \left\{ (s < p') \;\; = o \right\} \end{array}$$

[1] The negation of a proposition may also be represented by the symbol $\not<$. Thus $a \not< b$ is the contradictory of $a < b$. We have, accordingly, the following equivalent expressions:—I $(s \not< p') = (s < p')' = \left\{ (s < p') = o \right\}$
O $(s \not< p) = (s < p)' = \left\{ (s < p) = o \right\}$

$$
\begin{array}{l}
(s < p) \quad < \{(s < p') = o\} \\
(s < p') \quad < \{(s < p) = o\} \\
\{(s < p')' = o\} < (s < p)' \\
\{(s < p)' = o\} < (s < p')' \\
(s < p) \quad < \quad (s < p')' \\
(s < p') \quad < \quad (s < p)' \\
\{(s < p')' = o\} < \{(s < p) = o\} \\
\{(s < p)' = o\} < \{(s < p') = o\}
\end{array}
$$

The cases in which no valid inference is possible have been omitted. It is understood that equating any proposition to o, e.g., $(s < p) = o$, means that the given proposition is *false*. On the other hand, a proposition which is not equated to o is understood to be taken as true "for the sake of the argument"; in other words, such a proposition is understood to be equal to 1, the symbol of truth.

(2) The relations ordinarily included under *obversion* are these:

$$
\begin{array}{ll}
(s < p) = (s < p'') & (s < p')' = (s < p''')' \\
(s < p') = (s < p''') & (s < p)' = (s < p'')'
\end{array}
$$

In the expressions, of course, '' stands for the negative of the negative of the term in question, while ''' stands for the negative of the negative of the negative. It is manifest, however, that in Symbolic Logic obversion has little or no importance. In ordinary discourse a negative statement may sometimes have greater rhetorical effectiveness than an affirmative statement; and it is therefore a matter of considerable importance to know which forms of expression are equivalent. In Symbolic Logic we recognize at once *that* $a'''''' = a'''' = a'' = a$ and that $a'''''' = a''' = a'$; or, in general, that an *even* number of negations is equivalent to an affirmation, while an *odd* number of negations has the force of one negation.

(3) Of much greater importance is the *law of contra-*

position: $(s < p) = (p' < s')$. This means that if the class s is included within the class p, then everything in the universe which is outside of p is included within that part of the universe which is outside of s. Or, if proposition s implies proposition p, it follows that the contradictory of s implies the contradictory of p, etc. Observe, however, that in Symbolic Logic we need not pay attention to more than one contrapositive of a given inclusion or implication; for the other contrapositive would be merely the obverse of this, i.e., would differ from it only by the addition of *two* signs of negation. The relations which fall under contraposition are, then, the following:

$$\begin{aligned}(s < p) &= (p' < s')\\ (s < p)' &= (p' < s')'\\ (s < p') &= (p < s')\\ (s < p')' &= (p < s')'\end{aligned}$$

The first two equations represent the contraposition of propositions A and O. The last two represent what is in effect the simple *conversion* of E and I. In other words, what is known in ordinary logic as conversion is here seen to fall under contraposition. The exception to this statement is *conversion by limitation.* It is a one-way inference and may be symbolized thus: $(s < p) < (p < s')'$.

Complications.—Difficulties arise, however, in translating the operations of immediate inference into the Symbolic Logic, unless we impose certain restrictions on the meaning of our terms. To avoid these difficulties we assume that no term becomes 1 or 0. For (1) suppose the predicate to be 1 or "universe" in the proposition "All A is B." It is obvious that the proposition is true. But from this proposition, by converting its full contraposition, we derive "Some non-A is non-B." This, however, cannot be true; for if B is 1, non-B is 0. (Of course, if A is also 1, non-A will

be o, and we shall have $o < o$; but this need not be the case.) Again (2) suppose the *subject* of an A proposition to be o. Then we may symbolize it by $o < x$. The corresponding E proposition will be $o < x'$. But since the null class is contained in *any* class, *both* contraries are true. And if the principle of contradiction holds, *both* sub-contraries are false, and the relation of subalternation must be given up.

Other difficulties may be found, if we search for them. Our examples may be expressed in symbols as follows:

$$(1)\ \ (s < p)(p = 1)' < (s' < p)'$$
$$(2)\ \ (s < p)(s = o)' < (s\ \ < p')'$$

The latter of these may be called the *postulate of subalternation;* the former, the *postulate of inversion.* These postulates are indispensable in the traditional logic, and make it a special system within the wider system of logic in general.

Syllogisms.—In syllogistic reasoning a conclusion is said to follow from two premises. The two premises are simultaneously affirmed. Accordingly, from the point of view of Symbolic Logic, we may say that the conclusion is implied by the *product* of the premises. For example,

$$\{(a < b) < (c < d)\}\ (a < b) < (c < d)$$

is a hypothetical syllogism; while

$$\{(a < b) + (c < d)\}\ (a < b') < (c < d)$$

is an alternative syllogism. In the same way the following is an example of the combination of hypothetical and alternative reasoning which is called the dilemma:—

$$\{[(a < b) < (c < d)]\ [(e < f) < (g < h)]\}$$
$$\{(a < b) + (e < f)\} < \{(c < d) + (g < h)\}$$
$$\{(a < b) < (c < d)\}\ \{(c < d) < (e < f)\}$$
$$\{(e < f) < (g < h)\} < \{(a < b) < (g < h)\}$$

is a hypothetical sorites.

$$\{(a < b)\ (b < c)\ (c < d)\ (d < e)\} < (a < e)$$

is a categorical sorites.

$$(a < b)\ (b < c) < (a < c)$$

is, of course, a categorical syllogism.

The distinctions of moods and figures are of very little importance in Symbolic Logic. It is, however, very easy to represent any mood of any figure. Thus

$(m < p)\ \ (s < m)\ \ < (s < p)$ is AAA of Figure I.
$(p < m)\ \ (s < m')\ \ < (s < p')$ is AEE of Figure II.
$(m < p)\ \ (m < s')' < (s < p')'$ is AII of Figure III.
$(p < m')\ \ (m < s')' < (s < p)'$ is EIO of Figure IV.

Applying the "Principle of the Syllogism." [1]—What is the relation of the valid moods to the principle of the syllogism, $(a < b)\ (b < c) < (a < c)$? The expression for AAA of Figure I corresponds most nearly to the "principle." Indeed, the only difference is that in the "principle of the syllogism" the *minor* premise appears first, while we have been accustomed to write the major premise first. Let us then, first of all, in testing a syllogism by reference to the principle, arrange the propositions in this order: minor premise, major premise, conclusion. This done, any valid syllogism having a universal affirmative conclusion will be found to be identical with the "principle."

Let us next consider syllogisms having a universal negative conclusion. For example, take the second figure syllogism, $(s < m)\ (p < m')\ (s < p')$. This seems to be at variance with the principle. The order of terms in the second proposition is reversed, and we have m' instead of m. But apply the law of contraposition, and all comes right. For if we contrapose the second proposition, the entire expression becomes $(s < m)\ (m < p') < (s < p')$. And this corresponds exactly with the principle.

[1] See page 120.

In the third place, let us consider the case of a syllogism which has a particular conclusion, as for example, $(m < p)\ (m < s')' < (s < p')'$. Here we use an indirect method of proof. *If we can show that the contradictory of the conclusion is at variance with the premises, we then know that the conclusion is implied by them.* Now the contradictory of the conclusion is in this case $(s < p')$. Let us then take this as our first premise. We then have, $(s < p')\ (m < p)$. Contraposing the second proposition this becomes, $(s < p')\ (p' < m') < (s < m')$. Contraposing this result we obtain $(m < s')$ which is the contradictory of the second premise of the syllogism as originally given. Therefore the original conclusion follows from the original premises.

In general, then, if the conclusion is negative and universal, it should be possible to reduce it to the typical form by contraposing one of the premises. And if the conclusion is particular, it should be possible to show that, assuming one of the premises to be true, the contradictory of the conclusion may be shown to be at variance with the other premise.

The Superiority of Symbolic Logic.—In what respects is Symbolic Logic superior to the ordinary logic? Several points of superiority may be suggested.

(1) Symbolic Logic has the advantage of *greater compactness.* Its symbols can be written in less time than the words of ordinary language, and after the writing is completed, the eye can take in much more at a glance.

(2) Symbolic Logic is capable of *greater precision.* Two outstanding instances may be mentioned. The first is the avoidance of ambiguity in propositions of the type, "All S is P." As already remarked,[1] we may distinguish two senses of this proposition. One of them corresponds to the expression $s < p$, and the other to the expression $s = p$.

[1] See page 79.

When the proposition is expressed in the form "All S is P," unless we have some further information concerning the entities denoted by S and P, we can not be sure which meaning is intended. Since Symbolic Logic has the two distinct forms, there is no room for misunderstanding.

The alternative proposition, too, as we have seen,[1] is open to misunderstanding. Unless we have more information than is given by the proposition itself, we can not be sure whether it is meant to express a disjunction, or not. In Symbolic Logic, however, the distinction between a complete disjunction and a mere alternative is clearly indicated. The latter case, in which the alternatives are not understood to be mutually exclusive, is represented by the simple logical sum, $a + b + c$, etc. On the other hand, the case in which the alternatives are understood to be mutually exclusive is represented by the "disjunctive sum, $a + a'b + a'b'c$, etc. In this way the universe of discourse may be divided exhaustively into parts which do not overlap. This may be represented as follows:

$$1 = a + a';$$
$$1 = ab + ab' + a'b + a'b';$$
$$1 = abc + abc' + ab'c + ab'c' + a'bc + a'b'c + a'b'c';$$

and so on. This is the sort of division known as *dichotomy*,[2] that is the whole is divided into *two* parts, each of these parts into two, each of these into two, and so on. Thus in our representation, the universe is divided first into that which is a and that which is *not-a*. That which is a is then subdivided into that which is b and that which is not b, and that which is *not-a* is subdivided in the same way. In the third place, each of the four subdivisions thus far obtained is divided into a part which is c and a part which is *not-c*.

[1] See page 25.
[2] See page 229.

(3) In Symbolic Logic *the copula is generalized* so that inferences may be based upon relations some of which can hardly be expressed in the syllogism. Consider, for example, the following arguments:

A is better than B.	Noah is an ancestor of Abraham.
B is better than C.	Abraham is an ancestor of Moses.
∴ A is better than C.	∴ Noah is an ancestor of Moses.

It is obvious that these are sound arguments. Nevertheless each of them violates the rule of the syllogism which forbids four terms. Thus the predicate of the first premise of the one argument is the term *better than B,* but the subject of the other premise is simply *B.* There is accordingly no *middle term.* If, however, we consider the phrase *better than* to belong, not to the predicate, but rather to the copula, the difficulty disappears. This, then, is what we mean by the generalization of the copula: the symbol $<$ may be interpreted so as to mean, not simly *is* or *is included within,* but *better than, ancestor of, north of,* etc., and the principle of the syllogism, $(a < b)\ (b < c) < (a < c)$, will still hold good.[1] The relations denoted by such phrases as these are

[1] It may, perhaps, be objected that when $<$ is given the meaning *is included within,* the principle of the syllogism is self-evident; but that when the symbol stands for some other transitive relation, an assumption, a hidden premise, is presupposed. For example, consider again the "syllogism," "A is better than B, and B is better than C; therefore A is better than C." It may be urged that another premise is presupposed, namely, "If it be true that B is better than C, then whatever is better than B is better than C." The objection is, however, based upon a misconception. For it is just as necessary, and just as needless, to supply such a premise in the case of the ordinary syllogism, which is based upon the relation *is* or is *included within,* as it is in the case in question. Indeed, the proposition, "Whatever is better than B is better than C," is not something added to "B is better than C"; the former is rather an interpretation of the latter. And, as we saw in our discussion of the categorical syllogism, any proposition of the form *S is P* may conveniently be interpreted as, "Whatever is S is P," or "If x is S, then x is P." In general, then, *whatever transitive relation* is represented (in the primary propositions) by $<$, $(a < b) = (x < a) < (x < b)$. (It must be remembered, however, that the copula of a *secondary* proposition is always *implies* or *it follows that.*) In the same way, the "principle of the syllogism," may be expanded thus:

$$\{(a < b)\ (b < c) < (a < c)\} = \{(a < b)\ [\,(a < b) < (a < c)\,]\} < (a < c)$$

known as *transitive relations.* There are, however, certain relations which can not be made the basis for an inference. The phrases by which they are denoted cannot be used as the copula in an argument. These are known as *intransitive relations.* Examples of this latter class of relations are *cousin of, distinct from, at enmity with,* and the like. Our point is, then, that Symbolic Logic has the advantage of being able to represent *any transitive relation* by the symbol $<$, and to proceed as if $<$ had the more usual meaning of *is* or *included within.*

(4) Finally, Symbolic Logic has greater power than the ordinary logic because it makes use of transformations of which the ordinary logic knows nothing. Consider, for example, the formulas of DeMorgan, which greatly facilitate the finding of the negative of a sum or of a product. These formulas are as follows:

$(a+b)' = a'b'$	$(ab)' = a' + b'$
$(a+b+c)' = a'b'c'$	$(abc)' = a' + b' + c'$
Etc.	Etc.

We shall not attempt a demonstration of these formulas; but they may be shown to hold for areas, if overlapping figures are drawn to represent *a, b, c,* etc.

Another example of the power of Symbolic Logic is afforded by its doctrine of "functions." The limits of an elementary textbook do not permit a full discussion of this topic. Suffice it to say that expressions of the form, $ax + bx' = o$ are called "developed functions of x." Since in Symbolic Logic there are no exponents and no numerical coefficients, any function of x may be reduced to this form. The factors associated respectively with x and x' are then represented by a and b, which may be called the coefficients of x and x'. Functions developed in this manner possess the following properties:

"The sum or product of two functions developed with respect to the same letters is obtained simply by finding the sum or the product of their coefficients. The negative of a developed function is obtained simply by replacing the coefficients of its development by their negatives." (Couturat, *Algebra of Logic,* p. 34f.)[1]

Historical Note.—I shall close this account of Symbolic Logic with a brief historical note. In the thought of the philosopher Leibniz (1646-1716), Symbolic Logic grew out of an interest in the idea of a universal language. Leibniz believed that all thought might be analyzed into a relatively small number of elementary ideas, and that each of these ideas might then be represented by a distinctive character. These characters, or "ideographs," would be the same the world over, just as the "arabic" numerals are employed universally.

Another example of such ideographs is the system of symbols employed in chemistry. As part of his projected universal language, or *"characteristica universalis,"* as he called it, Leibniz developed a *"calculus ratiocinator,"* in which he anticipated the fundamentals of our modern "Symbolic Logic." Leibniz, however, did not publish his results, and the real founder of Symbolic Logic was George Boole (1815-1864). Boole was a mathematician who conceived the idea of transforming the traditional logic into a kind of algebra. In carrying out this idea he rediscovered many of the theorems of Leibniz. After Boole, many investigators have sought to develop and improve his work. One of the most important of these was Ernst Schroeder. And the type of Symbolic Logic which has been sketched in this chapter is commonly referred to as the "Boole-Schroeder Algebra."

[1] The reader will understand, of course, that this chapter does not pretend to give a complete account of Symbolic Logic. We have only scratched the surface.

Exercises.—1. Express in symbols.

(a) Nos. 1, 2, 5, 10, 12, and 13 on p. III. 8.

(b) Nos. 7, 12, 14, and 15 on p. V. 7.

2. Write an original hypothetical syllogism, an original alternative syllogism, an original categorical syllogism, an original dilemma, and an original sorites. Then express each in symbols.

3. Interpret the following:

(a) $(a < b)\ (c < d)\ (e < f) < (m < n)$

(b) $(a = b)\ (b = c) < (a = c)$

4. Express in symbols:

(a) X is now a Federal judge. Therefore he must have been appointed by the President, and this appointment must have been approved by two thirds of the Senate.

(b) "No person shall be a representative who shall not have attained to the age of twenty-five years, and been seven years a citizen of the United States, and who shall not, when elected be an inhabitant of that State in which he shall be chosen."

(c) In order that a bill may become a law it must receive at least a majority vote of the House of Representatives and also of the Senate, and must either be approved by the President or, if vetoed by him, must be passed over his veto by a two thirds vote of each house.

(d) "Treason against the United States shall consist only in levying war against them, or in adhering to their enemies, giving them aid and comfort."

(e) "The Congress, whenever two thirds of both houses shall deem it necessary, shall propose amendments to this Constitution, or, on the application of the Legislatures of two thirds of the several States, shall call a convention for proposing amendments, which, in either case, shall be valid to all intents and purposes, as part of this Constitution, when ratified by the Legislatures of the several States, or by conventions in three fourths thereof, as the one or the other mode of ratification may be proposed by the Congress."

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CHAPTER VIII

HYPOTHESES AND THEIR USE

The Search for Premises.—Up to this point we have been chiefly concerned with the formal validity of arguments. It has been our purpose to discover rules by which various types of argument may be shown to be equivalent to forms which are admittedly valid. A syllogism, for example, as we have pointed out repeatedly, may be valid even though none of the propositions composing it be true; on the other hand, though every proposition be true, the syllogism may nevertheless be invalid. All that is meant when a syllogism is said to be valid is that, given true premises, it inevitably leads to a true conclusion. But where shall we get our true premises? Shall we treat each of them as the conclusion of an antecedent syllogism? In that case it will be necessary to seek premises to support them, and of course other premises to support *these,* and so on *ad infinitum.* It is manifest that in this way we simply postpone and multiply the difficulty.

We cannot then verify our premises by any of the forms of inference hitherto discussed; not by syllogistic reasoning, and not by any form of so-called immediate inference, because for every premise we must seek a premise. One way in which propositions may be secured is by the method of hypotheses. This is *par excellence* the method employed in the natural sciences. Let us now try to make clear what is meant by a hypothesis, and how a hypothesis may be verified.

Exercises.—An Approach to the Notion of a Hypothesis.—

1. If you were asked to guess which side of a coin would fall uppermost, would you have anything upon which to base the guess, or would it be wholly without intelligent ground? (It is assumed, of course, that the coin is "fair," and that it is properly tossed.)

2. A little child says to her father, "I have a big piece of candy for you. Guess which hand it is in." Upon what fact or facts might the father base his "guess"?

3. Can you think of a sentence which fulfills the following requirements?

(a) It is a popular maxim, which is meant to encourage thrift.

(b) It consists of five words, four of which have the same initial letter.

(c) Two of these words are nouns, and two are adjectives.

4. An old-fashioned merchant marks the selling-price of his goods in ordinary numerals, but the cost in accordance with a key or cipher. This is a single word consisting of nine letters. The first letter represents 1, the second 2, and so on. The letter 0, which is not found in the key word, indicates zero. Discover the key word, assuming that certain articles have been marked as follows:

S. P.	Cost	S. P.	Cost	S. P.	Cost
$ 1.50	f r a	$ 6.00	g t o	$10.50	t r o
10.00	i l o	11.00	y r a	8.00	l a o
3.00	r g o	4.25	u a o	10.75	t r a
7.00	a i a	5.50	g f a	14.00	y t o

It may help you in your search if you are told, further, that the word for which you are looking is a synonym of the word *economy*.

5. Valuable property was stolen from the mansion of Mr. X. The morning after the robbery footprints were discovered leading from the mansion to a nearby hovel. The occupant of the hovel, a solitary old man who had occasionally been employed about the house and grounds of Mr. X, was found fast asleep. Several silver spoons bearing the monogram of Mr. X were discovered in a cupboard which stood beside the old man's cot. When accused of the crime he, however, protested that he knew

nothing about the spoons. He declared that at the time the robbery was said to have been committed he was twenty miles away, at the home of a brother; and at the hearing his brother testified that the accused had remained with him until three o'clock in the morning. The old man had borne a good character, except for the fact that he had once served time in the county prison for assaulting a peddler upon the occasion of a New Year's jollification. However, in the twenty years which had elapsed since his release from prison, he was supposed by all who knew him to have lived a sober and upright life.

Assuming the correctness of the facts just given, do you think that this man was guilty? If not, what other hypothesis would you suggest? If you were the prosecuting attorney, what additional data would you seek, in order to make the web of circumstantial evidence more complete.

6. Do you think that the earth is flat, or that it is round? To what facts would you appeal in support of the hypothesis which you adopt?

7. You probably know that toward the spring of the year potatoes are likely to "sprout." Most of the sprouts are white throughout their entire length, but those which chance to grow through a hole in the bag or box in which the potatoes are kept are often found to be green at the tips. What hypothesis would you suggest to account for this difference? What other facts can you think of which support this hypothesis?

Definition of the Term Hypothesis.—A hypothesis is any concept or supposition which is used to organize a group of facts. In so far as the hypothesis conforms with the facts it is said to be *verified;* and a fact which is in agreement with a hypothesis is said to be *explained* by that hypothesis. Synonyms of the word hypothesis are *conjecture, opinion, view, theory, doctrine,* etc. In other words, a hypothesis is regarded as successful or valuable in the proportion that it is found to conform with the relevant facts; and perhaps no better preliminary definition can be found than this, that a hypothesis is any supposition which is offered as a *candidate for verification.*

Two Contrasted Ideals: Contemplation vs. Control.—This doctrine of hypotheses is characteristic of the modern scientific view of the world. For Parmenides, and Plato, however, as also for most mediæval thinkers, ideas or general conceptions, were not hypotheses in the sense in which we have just described the latter. For these thinkers, genuine reality was found, not in the realm of particular facts, certainly not in the realm of sense-perception, but rather in the realm of general ideas. The idea was the true reality of which the individual thing was an imperfect manifestation. Thus, according to Plato, the flower is white because it "participates" in *whiteness;* the young man is beautiful because he "participates" in *Beauty.* That is to say, the generic idea is logically prior to its particular exemplifications. For Parmenides, the world of sense-perception, the world of color, sound, motion, change, is an illusion; the true reality can be known only to pure reason. Consequently if ideas are worthy, beautiful, or in any sense precious as viewed by Reason, it is wholly irrelevant to inquire whether they are in accordance with the facts. Our ideas do not agree with the facts? Well then, these thinkers would say, So much the worse for the facts. In other words, for the Parmenidean, Platonic, or mediæval mind, the true, the real idea is the one which is valuable as an object of contemplation. In opposition to the ideal of *contemplation,* the modern mind sets the ideal of *control.* The mediæval view of ideas is artistic; the modern is utilitarian. The modern man constructs general ideas or principles, not that he may stand off and admire them, but that he may use them in transforming the environment in accordance with his needs or in adjusting himself to his environment.

Fact, Theory, and Working Hypothesis.—The value of a hypothesis consists entirely, then, in the success with which it explains facts. Other things being equal, as we shall

see, the hypothesis which explains the greater number of facts is the better hypothesis. On the other hand, to explain a fact means nôthing more than to bring it, along with other facts under some hypothesis, to show that it and they are all logically implied by a general principle. And the greater the assemblage of facts thus brought together into one system, the more satisfactorily each of these facts has been explained. In some cases, however, two or more hypotheses are equally successful in explaining the facts. This means that the data now available are not sufficient to justify a decision. Both or all of these hypotheses may then be retained as "working hypotheses." There may, indeed, be facts which are inexplicable on any hypothesis that has been suggested. Where all the facts are explained satisfactorily by two or more hypotheses, or where there remain facts which are not explained by any, the wise man keeps an open mind, he does not commit himself to any hypothesis, but makes use of all as guides in his further investigation.

It is important to observe, however, that these working hypotheses differ only in degree of certainty from so-called established knowledge. The only difference between a working hypothesis and a theory, on the one hand, and between a theory and a fact, on the other, is in relative completeness of verification. A *fact* is a hypothesis which is so well established that most of us never feel the need of proving it. A theory is not so well established as a fact; it is still a subject of controversy. Yet a theory is better established than a *working hypothesis,* which, as we have seen, is one of two or more rival hypotheses, of which we are not able to decide which is the best.

Just which hypotheses shall be called facts, which theories, and which "mere hypotheses," or working hypotheses must necessarily depend very largely upon the enlightenment or the prejudice of the individual mind. For example, evolu-

tion is regarded by some as only an "unverified hypothesis," while by others it is considered so well established that they call it a fact. Most of us would probably say that it is a fact that the earth is round and that it moves; a fact that material particles attract each other with a force which varies directly as their mass and inversely as the square of their distance. Yet there was a time when each of these was propounded as a new and startling hypothesis.

Percepts as Hypotheses.—It has no doubt seemed very strange to the learner that anyone should say that a *fact* is a hypothesis. Yet, strange as it may seem, this appears to be the most satisfactory position. While hypotheses depend upon facts for their verification, facts *themselves are but hypotheses of a more primitive sort.*

(1) This is most obviously true in the case of such facts as have been denied or questioned. You say that you saw a rabbit in the back yard. I suggest that you have made a mistake. You appeal to Henry for support, and Henry says that he saw it too. If I am still incredulous you take me to the yard and show me the tracks. Thus the fact of the rabbit's presence is verified by appealing to other facts.

(2) Again, I see that the rails of a railroad track converge at a distance, and that a stick which is half in air and half in water is bent at the surface of the water. Yet I do not call these facts, although I "see them with my own eyes"; I call them *illusions.* In other words, an illusion is a rejected hypothesis. The hypothesis that the rails converge is negated by the fact that a train has just passed over the track in safety; and by the further fact that if I walk to the place where they seemed to come together, I find them just as far apart there as anywhere else. The hypothesis that the stick is bent must be given up, because if I run my hand along it I fail to discover the angle, and if I re-

move it from the water or plunge it completely under the water it no longer appears bent even to the eye.

(3) In normal vision, too, when we "observe" the world round about us, we do not simply receive impressions from the objects of sense, but our minds are actively at work interpreting and harmonizing the data of sensation. The so-called "facts of experience" do not migrate, as it were, into our consciousness, but are themselves constructs of the perceiving mind. It is a commonplace of modern psychology that perception consists essentially in the *interpretation* of sense-data. What we call the "real" table, the "real" chair, the "real" flower, etc., may not be exactly like any of the visual experiences upon which the percept of each is based. Rather in each case the percept which is regarded as real is the hypothesis which most satisfactorily organizes our sensory experience.

To be sure, in most cases of perception the process is less conscious than in the verification of a hypothesis; and in many cases there is no evidence of such a process at all, since by much practice a certain sense-datum has become the "cue" which immediately suggests the completed percept. But if we study the manner in which we *learn* to perceive, and consider our experience with unfamiliar sense-material, in which we find ourselves correcting and revising our percepts, we see that the process is essentially the same as in the verification of a hypothesis. A genuine percept is such an organization of the data of sensation as "works." Because it works, it is said to be *real*. An illusion, on the other hand, is an unsuccessful attempt to organize our sense-data. Because it does not work, that is to say, does not fit in with the organized whole of experience, it is stigmatized as unreal.

Hypotheses and Circumstantial Evidence.—This digression into the field of psychology is intended not to justify, but rather to illustrate our view of hypotheses. Another

illustration is found in legal procedure, in the use of what is called "circumstantial evidence." It is interesting to compare the method of a scientist in pursuit of truth with that of a court engaged in the trial of a criminal case. Suppose that there has been a murder. No one can be found to testify that he saw the commission of the crime. Witnesses, however, appear to swear that they saw a certain man walking toward the home of the victim on the night of the murder; others that they saw the same man on the street toward morning of that same night. The murdered man had been stabbed, and when the body was discovered, a bloody knife was lying beside it. A tradesman testifies that he sold this knife to the man accused of the murder a few days before the crime. After the arrest of the accused man, his room was searched, and a valuable coat belonging to the murdered man was found concealed in the bottom of his trunk. The day after the murder the man now accused of the crime had deposited a considerable sum of money in a savings bank.

With such an array of evidence, the case seems sufficiently complete. No one saw the commission of the crime, the evidence is purely circumstantial; yet the guilt of the accused is established beyond reasonable doubt. Notice, however, that no particular bit of evidence taken by itself would be sufficient to convict. A man might have been walking toward the home of the murdered man a short while before the probable time of the murder, and still be innocent. A man might have been walking away from the scene of the tragedy shortly after the probable time of the murder, and still be innocent. There might have been some mistake about the knife; someone else might have had a knife of the same kind, or the tradesman might be mistaken in his identification. The possession of the stolen goods is not necessarily a proof of complicity in the theft; the real thief might have placed them in the man's trunk so as to divert suspicion from him-

self. The deposit in the savings bank might have been made possible by a year of frugality, or the money might have been a gift from a friend, or it might have been withdrawn from another bank. In short, each item in the array of evidence, if taken alone, has comparatively small evidential value; it creates at the most but a slight presumption of guilt. Nevertheless all taken together create a presumption of guilt which is well-nigh irresistible.

The Choice of Hypotheses.—The evidence for most scientific hypotheses is of the same circumstantial kind. That is to say, there is no one fact which would by itself justify our acceptance of the hypothesis. And yet in many cases the cumulative effect of the argument is ample to produce practical certainty in the opinion of all fair-minded persons. The argument for evolution is a case in point. By organic evolution, or "transformism" as it may be designated more accurately, is meant the doctrine that existing species of plants and animals have arisen out of earlier species by a process of growth or development. Opposed to transformism or evolution is the hypothesis of *special* or *separate creation.* According to this latter hypothesis, the rose bushes now existing have descended without essential change from an original rose bush which came from the hand of the Creator; the horses now existing have descended from an originally created pair of horses, which in all essential respects were like the present representatives of the equine species; the men now living are descended from an original man and woman, who were in all essential respects like the men and women of to-day. According to the evolutionist hypothesis, however, plants which the botanist regards as belonging to quite different species are the descendants of the same form of earlier plant life; diverse species of animals have sprung from a common ancestral type; and even man himself is a blood relative of the so-called "lower ani-

mals." Between these two hypotheses the issue is joined. We may assume, for the sake of the argument, that the idea of *creation* is compatible with either hypothesis. The question is then concerning the *method* of creation. Were the several species of plants and animals created separately, or were they derived by a process of growth from some originally created living form or forms?

The method of proof is in principle the same as that of circumstantial evidence. Believers in evolution appeal to several lines of evidence, such as the argument from comparative anatomy, the argument from paleontology, the argument from embryology, the argument from vestiges, and the argument from geographical distribution.

(1) *Comparative Anatomy.* If we see two men who look very much alike we begin to wonder whether they are brothers or perhaps cousins. In other words similarity suggests, but of course does not prove, a common ancestry. In the same way a common ancestry is suggested by the many resemblances which run through species which are superficially different. The argument from comparative anatomy, however, if it stood alone would not prove transformism. The defender of separate creation might hold that the species were created independently, but that the Creator made use, in all or many of them, of certain ideas or types of structure, in the same way, for example, that an architect may use the same general plan in designing a great number of different houses.

(2) *Paleontology.* Geologists tell us that the layers of stratified rock were laid down through the agency of water. The time required was many millions of years. If this be true it is obvious that the lowest layers must be the oldest and the highest the most recent. Now in many of these stratified rocks "fossils" are discovered. These are the petrified remains of plants and animals which grew in the

period when the rock was formed. Accordingly if these fossils are arranged in the order of the rocks in which they are found the arrangement may be assumed to be chronological. Now it is found that, as we pass from age to age of geological time there appears to be a progressive development in the structure of the fossils. The farther apart in geological time, the more unlike these extinct species of plants and animals seem to have been. And in some cases, as notably in that of the horse, it is possible to trace a fairly continuous series of forms connecting species now existing with quite diverse species of early life. To be sure, in many cases, there are gaps in the record which have not been bridged, as if pages and whole chapters of the book had been lost; and the argument from paleontology, taken by itself, would not be conclusive. But if it is taken in connection with the argument from comparative anatomy, the two arguments together create a strong presumption in favor of the doctrine of natural descent.

(3) *Embryology.* An unborn animal or human being is called an "embryo." Students of embryology have discovered two sets of facts which powerfully strengthen the argument for the hypothesis of transformism as over against that of a separate creation of species: (a) The younger the embryo of a given species the more difficult it is to distinguish it from the embryos of other species. This supports the argument for a common ancestry, based upon comparative anatomy. (b) Any given embryo appears in its development to pass through a series of stages corresponding to those through which its species has passed, *if the hypothesis of evolution be true.* To some extent, the embryo "recapitulates" the history of the species. There would appear to be no reason for this on the hypothesis of separate creation, but on the hypothesis of evolution it seems natural enough.

(4) *Vestiges.* Not only the embryo, but even the adult

individual may contain indications of the path over which the species has traveled. At least this is the interpretation of vestigial organs which is given by adherents of the evolutionist hypothesis. In the horse, for example, little hoofs sometimes appear on the side of the "leg." These are reminiscent of the toes found in the horse's ancestor, but which the horse has lost; for the horse has only one toe on each foot, while the type which the paleontologists regard as the ancestor of the horse had four or five. In the human organism the most notorious vestigial organ is, of course, the vermiform appendix, which apparently has no function in man, but which is homologous to a portion of the intestines which has an important function in certain of the lower animals.

(5) *Geographical Distribution.* Land masses such as Australia which have been cut off from the other continental areas since relatively remote geological times have a peculiar fauna and flora. That is, they possess many species which are unlike those of the other land masses. Just why this should be so, if species were separately created and have continued down to the present without essential modification, is apparently without explanation. On the evolutionist hypothesis, however, this dissimilarity of species is exactly what we should expect. For if land masses were detached quite early in geological time, so that migration was impossible, we should expect the evolutionary process to take a different course on different sides of such barriers.

Of the two rival hypotheses offered in explanation of the existing differences among species of plants and animals, the scientific world prefers the hypothesis of evolution or transformism to the hypothesis of separate creation. Can evolution be proved? Well, if "to prove" means to demonstrate beyond the possibility of any doubt, then it has not been proved. But if this is our idea of proof, *no* scientific theory

has been proved. For, as we shall point out later, no scientific theory can be proved beyond all possible doubt. All that we have a right to require in the case of any scientific generalization is that *a preponderance of probability* shall be established. This is all that can be done for the theory that the earth is round or for the theory that the earth moves; and this is all that can be done for the theory of organic evolution.

Principles Governing the Choice of Hypotheses.—It is sometimes said that of two rival hypotheses we choose the one which explains the greater number of facts. We ourselves have spoken of a hypothesis as a *candidate for verification;* and, to complete the figure, the facts might be considered as the votes or the voters which decide between the rival candidates. The analogy with processes of voting and principles of majority rule in a democracy is, however, somewhat misleading. It is not enough that the winning or "true" hypothesis explain *more* facts than any of its rivals. It should explain all which another explains, and in addition should explain some facts which the other does not explain. Moreover, the hypothesis which is chosen should explain the facts *as simply* as possible. These requirements may now be presented as two principles which govern the choice of hypotheses. The first may be called the "principle of superimposition," the second the "principle of simplicity."

(1) The Principle of Superimposition. This name is appropriate, because when we compare the two groups of facts which are explained respectively by the winning hypothesis and any of its rivals, the former group should include the latter and more *besides.* The relation may be represented diagrammatically by two circles, one of which completely covers the other and also extends beyond it. The one array of facts is related to the other, not as the class *Germans* to the class *Frenchmen,* but rather as the class *Europeans* or

the class *human beings* to the class *Frenchmen*. The principle of superimposition recognizes a difference in the relative importance of facts. The mere principle of majority rule is unsatisfactory for the reason that the hypothesis which explains the less numerous group of facts, might perhaps explain certain facts of such importance as to overbalance the numerical advantage of the other.

(2) The Principle of Simplicity. Almost any fact may be explained by a given hypothesis if we are allowed to make additional assumptions or qualifications. Thus the facts would be explained at the cost of an increase in the complexity of the hypothesis. If, then, we wish to organize a certain group of facts, it may be possible, if we are permitted to make assumptions or auxiliary hypotheses, to bring them all under either of two rival hypotheses. In the case of the one, however, many more of these auxiliary hypotheses are required than in the case of the latter. The latter explanation is therefore the simpler and should be chosen.

An Illustration of the Method of Hypotheses.—The principles governing the choice of hypotheses may be illustrated by a consideration of the rival claims of the Ptolemaic and the Copernican theories of the universe. According to the former, which is also known as the geocentric theory, the earth is the center of the universe, and all the fixed stars and planets, including the sun and moon, revolve about it. According to the Copernican or heliocentric theory, on the other hand, not the earth but the sun is the center of our part of the universe, which is therefore called the *solar system*. All of the planets, including the earth, revolve about the sun, and the earth rotates on its axis. Now we all prefer the Copernican to the Ptolemaic hypothesis; but upon what facts do we base our preference? It is surprising how few people are at all clear on this point. Apparently most people accept the heliocentric theory to-day on the authority of their

teachers, just as most people in the Middle Ages accepted the geocentric theory on the authority of *their* teachers.

Let us make a list of some of the facts which ought to be taken into account. Some of these can be explained about as well by one hypothesis as by the other. Some can be explained much more simply by the heliocentric hypothesis.

(1) The succession of day and night.

(2) The succession of the seasons.

(3) Eclipses of the sun and moon.

(4) The phases of the moon.

(5) Tides and tradewinds.

(6) The motions of the planets with respect to the "fixed stars."

(7) The fact that a weight dropped from a great height (as from a tower or in the shaft of a deep mine) falls a little to the east of the vertical.

(8) The fact that a pendulum set swinging in a north-and-south direction (in our latitude) appears to change its plane of vibration.

(9) The fact that a gyroscope appears to change its axis of rotation.

(10) The fact that the equatorial diameter of the earth is greater than the polar diameter.

(11) The fact that in the case of certain constellations the stars seem to be farther apart at certain dates than at the dates six months earlier or later.

The catalog of facts to be considered might, no doubt, be extended. These, however, will be sufficient for our purpose. The question is, Can these facts be explained better if we suppose the earth to be stationary, or if we suppose it to move? When challenged to say why they prefer the Copernican hypothesis, people frequently say that the succession of light and darkness proves that the earth rotates, and that the succession of seasons proves that it revolves around the

sun. If we stop to think, however, we soon see that day and night, the seasons, eclipses, and the changes of the moon, can be explained as simply on the old hypothesis as on the new. We have only to assume that the sun revolves about the earth in a sort of spiral, so as to rise a little farther north on each morning than on the preceding during the period from December to June, and a little farther south each morning from June to December, and all of these more obvious facts are accounted for. The first four facts of our list may then be explained by either of the rival hypotheses. The remaining seven, however, are explained exclusively by the hypothesis that the earth moves. Hence, by the principle of superimposition, this hypothesis ought to be chosen. For, while tides and winds can be partly explained on the assumption that the earth is stationary, there is an eastward drift of winds and waters which is accounted for most simply if we remember that the equatorial regions, if we assume the earth to rotate, must have a more rapid eastward motion than regions in higher latitudes, and that northward moving masses of air and water will therefore seem to come from the southwest. The motions of the planets with reference to the fixed stars could be explained by the old hypothesis, if they always moved in the same direction. But retrograde motions frequently occur, and these occasion difficulty. The explanation of the eastward deviation of a falling body is in principle, the same as the explanation of the tradewinds. The top of the tower or of the shaft moves eastward with a greater velocity than the bottom, and the falling object shares this greater velocity. The pendulum and gyroscope tend to maintain their plane of motion, while the earth moves. The greater diameter of the earth at the equator is accounted for as a result of the bulging produced by the centrifugal tendency occasioned by the earth's rotation. And, finally, the apparent displacement of the stars

is occasioned by the earth's motion with respect to the stars as it revolves around the sun. For it is obvious that if a star cluster is in or near the plane in which the earth revolves, the earth must be 180 millions of miles nearer to it on the 21st of December, let us say, than on the 21st of June, or *vice versa*. And as we approach it the stars must appear to spread out, just as the trees of a forest seem to draw apart when we come nearer.

Some of these latter facts might also be explained, however, if we permit ourselves to make additional assumptions. The case which is historically of the greatest interest is that of the motions of the planets with reference to the "fixed stars." We have said that the retrograde motions of the planets occasion difficulty if we try to account for them on the geocentric hypothesis. The mediaeval astronomers accounted for them, however, by means of the "theory of epicycles." An *epicycle,* as the name itself suggests, is a circle upon a circle. It was assumed that the planet did not revolve directly about the earth, but rather in the manner in which the moon may be said to revolve about the sun. In other words, a planet was said to revolve about a center, and this center to revolve about the earth. Now it is clear that this doctrine of epicycles would account for the retrogression of the planets. The objection to it is that it greatly complicates the main hypothesis. And as more accurate observations revealed more and more irregularities in the planetary motions, it was found necessary to suppose epicycles upon epicycles. Thus the Ptolemaic hypothesis became exceedingly complicated. So much so that a Spanish prince who was being taught the astronomy received in his day is said to have protested to his tutor, "If I had been present when the world was made, I should have seen to it that it was made more simply." This apocryphal protest illustrates the requirement of simplicity. For, granted that

with sufficient ingenuity the geocentric hypothesis might be made to explain everything which is explained by the heliocentric hypothesis (thus avoiding a decision under the principle of superimposition), the latter hypothesis must nevertheless be pronounced the winner because it is more simple.

Cases in Which the Appeal to an Additional Hypothesis Is Justified.—If the additional hypothesis is not created *"ad hoc,"* that is, is not invented solely to provide for the particular difficulty in question; but is itself a hypothesis which is genuinely required to explain a system of facts, its employment is perfectly legitimate, and it should not be considered as a mere auxiliary or subsidiary of the hypothesis in question. An instructive example is afforded by the manner in which science has overcome the popular mediæval objection to the hypothesis of a round earth. It was objected that if the earth were round objects on the other side would fall off; or, at any rate, the water of the oceans would all run to the under side. The answer, as every schoolboy knows, is that there really is no "under side"; for "down" is not a fixed direction, since objects, when they fall, always move toward the center of the earth. At first sight this may seem to be a mere complication of the hypothesis of the rotundity of the earth. In reality, it is a special application of the hypothesis of universal gravitation. And this hypothesis applies, not only to terrestrial phenomena, but is found to apply as well to the motion of the moon about the earth, and to that of all planets, satellites, comets, etc.

The Law of Parsimony.—We have seen that in deciding between rival hypotheses we are governed by two principles, namely the *principle of superimposition* and the *principle of simplicity.* From the standpoint of the universe as a whole, we may combine these two principles in the more general principle or *law of parsimony.* The scientist seeks

to reduce the first principles or fundamental hypotheses required for the explanation of *all* facts to the smallest possible number. The ideal goal of science would be the attainment of one all-embracing hypothesis. The human mind craves a "monistic" view of the universe. That is to say, it desires to find some one substance or principle in terms of which all phenomena may be understood, some one concept or principle from which all others may be deduced. It may be that this goal is in the nature of the case unattainable. Perhaps the mind of man will always have to be content with a larger or smaller number of independent principles. Nevertheless, the monistic view of the universe must needs be recognized as the ideal which is to be approached as nearly as possible. Accordingly, the scientist seeks to explain the greatest possible number of facts by means of the smallest possible number of principles. In other words, the scientist is parsimonious or stingy in the use of hypotheses. And our rule for the choice of hypotheses may be rephrased thus: *in choosing between two rival hypotheses, we prefer the one which, taken together with all the others required for the explanation of the sum-total of phenomena, gives us the simpler view of the universe as a whole.*

Occam's Razor.—The law of parsimony has been called "Occam's razor," because it was employed by William of Occam, a scholastic philosopher, who flourished about 1300 A. D. *"Entia non multiplicanda sunt,"* said William in his scholastic Latin, *"praeter necessitatem."* Literally translated this means that entities (i.e., laws, principles, hypotheses, assumptions) are not to be multiplied beyond necessity. In other words, the scientist, wishing to reduce his hypotheses to the smallest possible number, refuses to admit an additional hypothesis until he is fairly driven to it. An instructive example of the application of this principle of scientific method is afforded by the attitude of the orthodox psy-

chologist toward the hypothesis of telepathic communication. The claim is made that thought can be transferred directly from one mind to another. It has been suggested that our brains may radiate "thought waves," analogous to the ether vibrations which are the stimuli for brightness and color sensations; and that some brains, perhaps those which are properly *attuned* to ours, may receive these waves as thought. The hypothesis possesses a certain attractiveness; but as long as all the accredited facts can be accounted for by reference to sight, hearing, and other already recognized avenues of sense-experience, the psychologist would be disloyal to the fundamental canon of scientific method if he assumed the existence of an additional channel for the reception of cognitive experience. Too much stress should not be laid here upon our figure of "avenues" or "channels" of cognitive experience; but we are seeking merely to illustrate the principle: the psychologist does right in refusing to accept the hypothesis of telepathy unless and until he is required to do so by an accumulation of facts which can be scientifically verified. In the meantime, however, the idea of telepathy may wisely be employed as a *working hypothesis* to suggest experiments and to serve as a guide for observation.

Exercises.—1. Which of the following illustrate the "method of superimposition?" In each case, which hypothesis is the better?

(1) Hypothesis A explains facts m, n, o, p, q, r, s, and t; hypothesis B explains facts m, n, o, and p, but no others.

(2) M explains a, b, c, d, o, f, g, h, and l; while N explains d, e, f, g, h, i, j, k, l, m, n, o, p, and q. Neither explains any facts other than those specified.

(3) X explains a, b, c, d, e, t, g, h, i, j, k, and l; Y explains a, c, e, g, i, and k. Neither explains any except those specified.

2. Has the following hypothesis been established? The muscular action of laughter clears the system of the energizing substances which have been mobilized in various parts of the

body for the performance of other actions. If this be true, the first question that presents itself is, Why is the respiratory system utilized for such a clarifying purpose? Why do we not laugh with our feet and hands as well? Were laughter expressed with the hands, the monkey might fall from the tree and, if by the feet, man might fall to the ground. . . . In fact, laughter has the great advantage of utilizing a group of powerful muscles which can be readily spared without seriously interfering with the maintenance of posture. . . . Why the noise of laughter? In order that the products of excitation may be quickly and completely consumed, the powerful group of expiratory muscles must have some resistance against which they can exert themselves strongly, and at the same time provide for adequate respiratory exchange. The intermittent closure of the epiglottis serves this purpose admirably, just as the horizontal bars afford the resistance against which muscles may be exercised. The facial muscles are not in use for other purposes, hence their contractions will consume a little of the fuel. An audience excited by the words of an impassioned speaker undergoes a body-wide stimulation for action, all of which may be eliminated by laughter or by applause.

Let us test this hypothesis by some practical examples. The first is an incident that accidentally occurred in our laboratory during experiments on fear which were performed as follows: A keen, snappy fox terrier was completely muzzled by winding a broad strip of adhesive plaster around his jaw so as to include all but the nostrils. When this aggressive little terrier and the rabbit found themselves in close quarters each animal became completely governed by instinct; the rabbit crouched in fear, while the terrier, with all the ancestral assurance of seizing his prey, rushed upon the rabbit, his muzzle always glancing off and his attack ending in awkward failure.

This experiment was repeated many times and each time provoked the serious-minded scientific visitors who witnessed it to laughter. Why? Because the spectacle of a savage little terrier rushing upon an innocent rabbit as if to mangle it integrated the body of the onlooker with a strong desire to exert muscular action to prevent the cruelty. This integration caused a conversion of the potential energy in the brain-cells into kinetic energy, and there resulted a discharge into the

blood-stream of activating internal secretions for the purpose of producing muscular action. Instantly and unexpectedly the danger passed and the preparation for muscular action intended for use in the protection of the rabbit was not needed. This fuel was consumed by the neutral muscular action of laughter, which thus afforded relief.

A common example of the same nature is that encountered on the street when a pedestrian slips on a banana peel and, just as he is about to tumble, recovers his equilibrium. The onlookers secure relief from the integration to run to his rescue by laughing. On the other hand, should the same pedestrian fall and fracture his skull the motor integration of the onlookers would be consumed by rendering physical assistance—hence there would be no laughter. (Crile, *The Origin and Nature of the Emotions,* pp. 93ff.)

3. Criticise the following: Dr. Hodgson, reporting experiments on the ever-interesting Mrs. Piper, says that when "Phinuit" is using uninterruptedly the medium's voice, her hand may all the while be carrying on some active communication as from another personality, reminding one of the story of Caesar and his amanuenses. Flournoy's reported case of Hélène "Smith" exemplifies the same thing. While oral communications were being given, ostensibly from Mars, Flournoy was often able to obtain from the medium's left index finger comments and suggestions claiming to be from a mundane person named Leopold, *alias* Cagliostro. Such cases as these are different from the ordinary phenomena of forgetfulness. Different personalities, each with characteristic tricks of thought, are present at the same time. The consciousness itself seems cleft, and the one side appears to be ignorant of the occurrences in the other.

The spiritistic interpretation of these things would at once, of course, destroy their value as evidence for unconscious mental action. If "Leopold" be regarded as a distinct mind using Hélène's finger as a means of communication, Hélène's ignorance of his mental operations would not indicate that these processes were unconscious *for him.* It would simply be like our own ignorance of the present thoughts of the Emperor of Germany. But if we adopt the psychological point of view, and regard the various "personalities" in a case like Hélène's

as but different forms of the activity of one person, then we must acknowledge that in the end they may favor the doctrine of the unconscious; but since we are as yet ignorant of so many essential features in these abnormal phenomena, one may well hesitate to decide just what they do mean. We must wait for more light on the question whether the second personality's thoughts (if thoughts they be, and not some amazing trick of the nervous system) are indeed unknown to the primary personality at the time when the expressive movements are taking place. For the most part we have to depend on the subsequent recollection of the person, and a negative result here is quite compatible with a dim consciousness of the other personality's thoughts during their actual occurrence. It may be as in our dreams, where a number of different personalities occupy the stage at the same time, each representing a different point of view, each ignorant of the next move of his fellows, and yet there is nothing strictly unconscious nor any absolute cleft in consciousness, for all the *dramatis personae* are included in the larger single mind which is their theater. In Flournoy's report it is extremely suggestive that when "Leopold" was presumably in exclusive possession of Hélène's senses and reactions, and remarks were dropped by persons in the circle that would have offended the medium, she apparently did not hear them, but, after the trance, she showed by her conduct for weeks that the slighting words had not been lost upon her.

The more cautious position, then, would be to regard these cases as due to parallel and relatively disconnected streams of activity in the one mind. In all possibility they are not beyond the range of self-observation were the person in a condition favorable to introspection, or had he the training and interest needed for detecting the most obscure activity of his own mind. (Stratton, *Experimental Psychology and Culture,* pp. 76ff.)

4. John said: "I heard my bedroom clock strike yesterday, ten minutes before the first gun was fired. I did not count the strokes but I am sure it struck more than once, and I think it struck an odd number." John was out all morning, and his clock stopped at five to five the same afternoon. When do you think the first gun was fired? (Gates, *Psychology for Students of Education,* p. 350.) How does this illustrate the choice of hypotheses?

5. What facts can you think of which support the following hypotheses?

(1) That heat is a mode of motion.

(2) That matter is neither created nor destroyed.

(3) That lightning is electricity.

(4) That life always comes from life.

(5) That acquired traits are not inherited.

6. In the case of each of these hypotheses, formulate an alternative hypothesis.

7. In explanation of your choice between the hypotheses of each pair, indicate (a) what facts, if any, are explained satisfactorily by either hypothesis; (b) what facts are explained exclusively by the hypothesis which you prefer; and (c) what facts, if any, are explained exclusively by the other hypothesis.

8. Criticize and supplement the following discussions:

(1) For the sake of making my presentation specific, I shall state some of the important facts which the analysis into atoms explains;

(a) Pure substances combine in constant mass-proportions, no matter how small the quantities are which are worked with experimentally. This would be explained if larger masses are only multiples of some ultimate units between which the same mass-proportions hold.

(b) Certain substances combine in more than one proportion, but these proportions are rational or integral. This would be explained if there are parts between which rational ratios hold. But such ratios imply finite quantities—though these be very small.

(c) Certain substances combine with other substances in minimum ratios. This would be explained by the hypothesis that there are ultimate units which are the smallest that can enter into combination with the units of other elements.

(d) Substances which can be gasified exert pressure on the walls of the containing vessel,—the same pressure on each wall. For equal volumes this pressure varies directly with the temperature. These facts are explained by the hypothesis (1) that there are parts acting as wholes, in motion, and striking the walls of the vessel with a certain momentum, and (2) that the velocity of each part, and so the momentum, varies with the

temperature. These parts might be either ultimate units or simple multiples of these.

(e) Equal volumes of gases at the same temperature exert different pressures. This would be explained if the gases consist of moving parts,—the same in number in all gases of equal volumes at the same temperature,—which are of the same mass in any one gas, but of different mass in different gases. Then the velocity would be the same. These different masses might be in the same ratio as are the combining proportions of the substances in question.

(f) Equal volumes of gases at the same pressure are of different temperatures. This would be explained if the gas consists of moving parts, the same in number in all gases of equal volume at the same pressure, but with motions of different velocity. The masses would be the same.

(g) The data of (e) and (f) are together explainable *only* if in different gases at the same temperature and pressure both the masses and the velocities of the parts are different. The velocities must be if the masses are. And the masses must be —as 'combining proportions' show.

(h) That the volumes, pressures, and temperatures of two gases should be the same, but the masses and velocities different, is explainable, if equal volumes of gases, at the same temperature and pressure, containing the same number of parts, either ultimate parts, or complexes acting as units. (Avogadro's hypothesis.)

(i) That equal volumes of gases (in certain cases), at the same pressure and temperature, combine in definite proportions is explainable by the same hypothesis. (Spaulding, E. G., *The New Realism*, pp. 226f.)

(2) The flatfish changes its hue to conform to the color of the background on which the creature happens to lie. For a time biologists supposed that this adaptation was effected by some direct photochemism upon the skin. But a quarter century ago, Pouchet proved that it was brought about through the functioning of the eye. He found that blinded fish do not change their color adaptively. . . . Sumner has shown that the animal can adapt while its entire body (except for the eyes) is buried in sand or completely masked with a cloth or deeply stained. . . . Sumner prepared a number of back-

grounds, some reproducing various types of natural sea-bottom (fine sand, coarse sand, fine gravel, coarse gravel, of various colors), and some highly unnatural geometrical patterns (checkerboard, polka dot, stripes, screen, etc.). Placed in a tank having one of these patterns on its bottom, the flatfish began to copy the pattern *on its back*. . . . The time required to complete the imitation varied . . . from a few seconds to several days. (Pitkin, W. B., *The New Realism,* pp. 398ff.)

(3) There is the possibility that the earth carries along with it in its flight through space a sort of atmosphere of ether as it does of air. We must first get rid of this possibility by a preliminary experiment to see if a swiftly moving mass of matter does catch up and carry along with it a little of the ether. This would cause a sort of an eddy or disturbance in the ether in the neighborhood of the moving mass as a boat disturbs the water. For instance, a ray of light passing close to a rapidly revolving wheel would be a little deflected and show a distorted image. Sir Oliver Lodge tried this experiment and got negative results. That is, moving matter does not disturb or carry with it the ether. (Slosson, E. E., *Easy Lessons in Einstein,* p. 10.)

(4) That Einstein's conception of the universe is an improvement upon that of Newton is evidenced by the fact that Einstein's law explains all that Newton's law does, and also other facts which Newton's law is incapable of explaining. Among these may be mentioned the distortion of the oval orbits of planets round the sun (confirmed in the case of the planet Mercury), and the deviation of light rays in a gravitational field (confirmed by the English Solar Eclipse Expedition).

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CHAPTER IX

THE INDUCTIVE METHODS

Tradition Versus Observation and Experiment.—To us moderns it seems to be self-evident that theories or beliefs must stand or fall in accordance with their agreement or disagreement with the relevant *data.* The man who takes an attitude of "So much the worse for the facts" is laughed at. We all tacitly assume that principles are to be tested by reference to facts, rather than *vice versa.* This, however, has not seemed self-evident to all minds. For example, to Parmenides, a Greek who lived in the fifth century before Christ, it seemed self-evident that the precedence should be given to rational principles; and, accordingly, since the facts of motion and change seemed to be at variance with the principles of reason, he treated the facts as illusions. The fundamental principle of Hindu thought is essentially the same. Their sages teach that the "veil of Maya," that is to say of illusion, is thrown over all things. In mediaeval Europe, too, the typical schoolmen were far from awarding the primacy to facts. The "truths" which they most eagerly sought were valued, not for their utility in enabling man to adjust himself to this present world, but rather for their relevancy to the attainment of a better world hereafter. It was not, then, necessary that truth should conform to fact; that was true which had been promulgated by recognized authority. The way to learn the truths of theology was to go to the Scriptures, to the writings of the Fathers, to the living Church. In philosophy, instead of depending upon

their own rational powers, men were wont to ask, "What did Plato teach?" "What was the doctrine of Aristotle?" Even in a science so manifestly empirical as anatomy, most men were content to refer to the pages of Galen, a celebrated physician of the second century before Christ. Indeed, the dissection of the human body was long forbidden. So content were the men of that period with tradition that the *Geography* of Ptolemy was used as a textbook in the schools of Europe for fourteen centuries.

Induction and Deduction.—The beginning of the modern period was signalized by a change of attitude toward the authority of tradition. Men now began to say, with Francis Bacon, that the true "ancients" were the moderns, inasmuch as the latter have had a longer experience. The test of truth was no longer conformity to the authoritative teaching of the Church, or agreement with the great minds of antiquity, but rather conformity to the relevant facts. During the Middle Ages the method of thinking was in the main "deductive." Great stress was laid upon the use of the syllogism. Plenty of premises were at hand, to wit, the dogmas of the Church and the teachings of Aristotle; and with these at hand it was easy to proceed by syllogistic reasoning to the appropriate conclusion. Now, however, over against the deductive or syllogistic method of the mediæval schoolmen, the leaders of the new learning set the "new" method of *induction*.

It is important not to exaggerate the difference between these aspects of the reasoning process, as was done by Bacon and the other innovators. On the other hand, it is essential for the beginner to understand clearly the distinction between them. As usual, let us approach it by the help of examples. Suppose a man is trying for the first time to row a boat. There are two ways in which he may learn the relation between the motion of the oars and that of the

boat. On the one hand, he may recall what he learned in physics about the inertia of liquids and the action of levers. On the other hand, he may make random motions and see what happens. Whichever method is adopted, he will of course discover that when he pulls on the oars the boat will move in a direction opposite to that in which he is looking. The former method of reaching this result—by inference from knowledge already in his possession—is *de*ductive, the latter—by trial and observation—is *in*ductive. To take another example, a man wishes to know which way he must turn the wheel of an automobile to make the car turn to the right. He might solve the problem deductively by examining the steering apparatus and applying his knowledge of mechanics; or he might solve it inductively by actually turning the wheel and noting the effect on the motion of the car. In short, then, induction is the method of discovery by observation and experiment; while deduction is the method of using the knowledge which one has already discovered, or has accepted on the authority of some other investigator.

Two Methods of Proving a Proposition.—There are then two ways in which one may attempt to establish the truth of any given proposition; or, more accurately expressed, there are two *directions* in which the inferential process may move. This difference is sometimes described by saying that in induction we argue from the particular to the general, and in deduction from the general to the particular. For example, let us prove in first one and then the other way that *heat expands steel rails*. This is manifestly a universal affirmative proposition. It may be put in the form, "All steel rails expand as their temperature increases," or, "All cases of increasing temperature are cases of the expansion of steel rails." It may be established deductively by a syllogism such as this:

All things which are made of metal are expanded by heat;
Steel rails are things which are made of metal.
Therefore they are expanded by heat.

The syllogism could of course be expressed just as well in the hypothetical form. To establish the same proposition inductively, it would be necessary to examine the behavior of a sufficient number of steel rails, the temperature of which is known to have increased. Measurements should be made of the same rail at different temperatures, as might readily be done at a railroad track at different hours of the day, or in a blacksmith shop where bars of steel are heated artificially. The reader may now test his understanding of these two methods of proof by establishing, first deductively and then inductively, each of the following:

1. The admission of air into the pipe of the furnace between the fire and the chimney will check the fire.
2. The application of wood ashes will stimulate the growth of clover.
3. If smoke falls there is likely to be rain.

Deduction Depends upon Induction.—The terms "induction" and "deduction" are somewhat unfortunate. They suggest a more complete contrast than really exists. Indeed, as we have already suggested, they are not so much distinct kinds of reasoning as phases or aspects of one and the same reasoning process. It is readily apparent that deduction presupposes induction. How else should we obtain our premises? Only those who are content to rest upon *authority* or who imagine themselves to be in possession of universal principles whose truth is *self-evident,* can suppose themselves able to get along without observation and experiment. But in a world where authorities disagree it seems a rather violent assumption to regard any one of them as *infallible;* and since "intuitions" in like manner disagree,

and we do not appear to have any faculty by which to distinguish the self-evident principles which are false from those which are true, "self-evidence" cannot be regarded as an absolutely trustworthy test of truth. (It may be objected that we may obtain our first principles by the method of hypothesis rather than by induction, and we shall show further on that the "inductive methods" themselves consist essentially in devices for the elimination of hypotheses. But the term "induction" is usually employed in a sense sufficiently broad to include the method of hypotheses.)

Induction Depends upon Deduction.—That induction presupposes deduction may at first sight be less evident. Yet it is easy to show that it does. (1) The facts upon which the induction is based must be *relevant*. That is to say, there must be some means of circumscribing the area of the investigation; and this is possible only as the investigator already has in mind some idea of the result which is likely to be reached. But it is evident that the use of such a tentative idea or preliminary hypothesis involves deduction. (2) Many facts which ought to be taken into account are likely to be overlooked if there is no such preliminary hypothesis; for in their study of *attention* psychologists have discovered that what one perceives depends very largely upon the "idea in mind."

Induction by Enumeration.—While induction is not a distinct type of thought, but rather an aspect of the one inductive-deductive thought-process, it is nevertheless important that we should study this aspect carefully. A good way of approach is to begin with what has been called "induction by simple enumeration." Suppose, then, an orchard of a thousand trees. A man wishes to determine the kind or kinds of fruit. He accordingly samples the fruit of tree after tree, and finds in each case that it is *Baldwin apple*. (Not to complicate the case unduly, we are ignoring the

possibility that the same tree may bear two or more kinds.) If in this way he examines every one of the thousand trees and finds each to be a Baldwin, he is of course justified in saying that *all* the trees in the orchard are Baldwins. Cases such as this, in which every individual is examined, have been called cases of "perfect induction." As an example of a proposition derived from perfect induction Jevons gives the assertion that "all the months of the year are of less length than thirty-two days." We know the names of the months, and can readily ascertain that each has less than thirty-two days. In the same way we might examine all the books of a library and discover that *all* are English books.

It has been objected, however, that a process such as this should not be called induction, because it is not *inference* at all in the strict sense of the term, but merely a *summing up* in one statement of individual items of knowledge. However, by whatever name we prefer to call the process, we shall probably agree that where it is feasible to examine every instance the result thus obtained is quite satisfactory. A more serious objection to "perfect induction" is the fact that in many cases it is obviously impossible to examine every individual. Indeed, the chief motive for the use of the inductive process is the desire that, having examined a reasonable number of individuals, we may then be able to affirm something concerning the members of the same class which *have not* been examined. In other words, we desire to reason from the known to the unknown. To return to our example of the orchard of a thousand trees. After examining, let us say, the fruit of a hundred trees and finding each one of these to be a Baldwin, what conclusion is justified? Certainly not that *all* the trees of the orchard are Baldwins; and yet, as the number of the trees examined increases, none having been found which are not Baldwin, the *probability* that all are Baldwins increases correspondingly.

The Selection of Instances.—Indeed, if the instances are properly selected, we shall soon be justified in feeling *practically certain* that all the trees of the orchard are Baldwins. The trees which are examined should be fair samples; that is to say, they should be so chosen that each may be regarded as the representative of a considerable group. An induction which is based upon a relatively small number of representative instances is more trustworthy than one which is based upon a multitude of instances which are examined just as they present themselves. In order that the instances may be representative they must differ from one another in some significant respect. If, then, instead of taking the trees one after the other as we come upon them in walking through the orchard, we divide the orchard into approximately equal sections and then take a tree for examination from each section, and find no apples which are not Baldwin, we should be justified in concluding that *all* the trees are Baldwins, even if the number actually examined is comparatively small.

Just what is meant, however, by the proposition, "All the trees of this orchard are Baldwins?" Probably *not* that there is no probability *what*soever of the discovery of a tree of some other variety; but, rather, that we are practically certain that any tree which is a-tree-of-this-orchard will be found upon examination to be a Baldwin. In other words, we have discovered a connection between two groups of qualities;—between the qualities connoted by the phrase, "tree-of-this-orchard," and those connoted by the phrase, "Baldwin-apple-tree."

Sign and Cause.—We are all intensely interested in the discovery of such connections between various parts of our world. If we did not have this interest our mundane existence would be short indeed; for if the human organism is to survive, it must be able to adjust itself to its environment.

This adjustment involves two complementary processes: (1) the organism changes so as to adapt itself to the environment; and (2) the environment is changed so as to fit it to the needs of the organism. Now in order to adapt itself successfully to its environment, which is changing from month to month and from day to day, the organism must be able *to predict* the changes which are likely to occur; and in order to conform the environment to its needs, the organism must be able to control the course of events in the physical world. For example, the organism requires a certain temperature. In order to be warm it may consume a greater quantity of food, or it may build a fire. But in either case,—whether the organism is modified to suit the environment, or the environment is modified to fit the organism,—the cold weather must be foreseen, so that extra food and fuel may be provided; and if the temperature is to be increased artificially, there must be knowledge of the relation of fire to heat. These two needs, then, *the need to predict* and *the need to control* furnish the practical motive for natural science.

If, now, a connection has been discovered between two things or events, as A and B, such that the presence of A enables us to predict the presence of B, A may be called the "sign" of B. Also, if we are able to produce or prevent the occurrence of A, and there exists an invariable connection between A and B, A is said to be the "cause" of B. It is obvious that all causes are signs; but some signs are not causes. Thus fire is a sign, and also a cause, of heat; while smoke is a sign, but not a cause, of fire; and day is a sign, but not a cause of night. In order, then, that we may treat things and events as signs and causes, the connections between the elements of our environment must be discovered. And this is the task of the inductive sciences. It is their business to provide us with trustworthy propositions con-

cerning the interconnections of natural phenomena. Thus we gain increasing control of our world.

Mill's Canons of Induction.—At its best, however, the inductive process gives us only a high degree of probability; the conclusions to which it leads are practically, not absolutely certain. In order to reduce to a minimum the likelihood of error, logicians have formulated certain rules or canons for the discovery of the causes of phenomena. These canons are usually quoted in the form in which they were given by John Stuart Mill. Several concrete examples may prepare the way for the more abstract presentation and discussion:

1. A housewife buys butter from several different creameries. Each make is put up in prints of various shapes and sizes. Some is artificially colored and some is not. She puts some of each make, size, shape, and color in a cupboard, some on a shelf in the pantry, and some she leaves standing on the kitchen table. In a few hours it all becomes soft. If this happens on an August morning, what would you suggest as the cause of the phenomenon?

2. The same housewife, not being completely satisfied with the result of the previous experiment, on another August morning cuts two pieces of butter from the same print, and puts one in the refrigerator, while the other is allowed to remain on the kitchen table. The latter soon becomes soft, while the former remains firm. What is the cause?

3. A piece of butter placed in a kitchen cupboard remained firm all night; but the next day as the temperature of the house increased the butter became softer and softer. What does this prove?

The answer in each case is of course that the butter is softened by heat. Now these three examples illustrate the most important of the inductive methods. The first illustrates the "method of agreement," which, as we shall see, is

a modification of the method, or lack of method, of "simple enumeration." The second illustrates the "method of difference," which is the method exemplified in many experimental procedures. The third is an example of the "method of concomitant variations." The name of this last is self-explanatory. It evidently means the study of variations or changes which accompany each other.

Observe carefully the essential structure of the argument in each of these examples of inductive inference. In which case are the samples of butter taken so as to be as *unlike* one another as possible? In which are they taken as nearly alike as possible? In which is the cause of the phenomenon a circumstance in which the instances *differ?* In which is it a circumstance in which they agree? In which is an increase in one phenomenon accompanied by an increase in the other.

In addition to these three methods, Mill speaks of two others, which he calls the "joint method of agreement and difference," and the "method of residues." These, however, are not independent methods, but are manifestly combinations of the method of agreement and the method of difference.

The Method of Agreement.—Mill's statement of the canon of this method is as follows: "If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon."

Let us guard against misconception. In popular usage the word "phenomenon" means an unusual or extraordinary event. Thus a "phenomenal downpour" is an unusually heavy shower; an "infant phenomenon" is a very remarkable child, etc. In philosophy and science, however, a "phenomenon" need not be anything remarkable; *any event what-*

soever is a phenomenon; or, more accurately, a phenomenon is an event which is thought of as occurring repeatedly. Can the same event happen twice? Probably not, if by the "same" is meant the *same in every respect.* Yet two or more occurrences which differ only in unimportant respects may be considered as repetitions of the same event. Each of these occurrences is then called an *instance,* and the event of which it is said to be an instance is called the *phenomenon.* For example, if typhoid fever is the phenomenon under investigation, every case of typhoid is an instance of the phenomenon. If the freezing of water is the phenomenon, every time a portion of water freezes we have an instance.

In each of the examples given in the preceding section, the softening of butter is the phenomenon in question. In the first, which is an example of the method of agreement, there are many instances of this phenomenon. All of these instances, much as they differ from one another in size, shape, color, etc., have *one* circumstance in common:—they have all been exposed to the heat. In accordance with Mill's canon we therefore infer that heat is the cause of the phenomenon.

Attention has often been called to the fact that Mill's statement of the canon of this method, if interpreted literally, makes an impossible requirement. He requires that the several instances of the phenomenon in question shall "have *only one* circumstance in common." Now it is obvious that many circumstances are always at least approximately the same. For example, all the pieces of butter were in the same house; all had been handled by the same person; all contain salt, etc. Some of these points of agreement, however, are unimportant, and may safely be disregarded. Nevertheless, we can rarely or never be perfectly sure that there is only one important point of agreement. Consequently, in the practical application of the method of agree-

ment all that can be expected is that the instances of the phenomenon shall be selected so as to differ among themselves *as much as possible*. In other words, the number of points of agreement, and therefore the number of the "circumstances" among which the cause must be sought, is reduced as much as possible.

As already pointed out, the method of agreement is a modification of the method of simple enumeration. In the latter method, the instances may be examined just as they present themselves; while in the former, instances are chosen which are as different from one another as it is possible to find them.

The Method of Difference.—The method of agreement, however, far superior as it is to the method of simple enumeration, cannot by itself be regarded as absolutely trustworthy. The results suggested by it require to be tested by the employment of the "method of difference." The canon of this method is expressed by Mill as follows: "If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon."

Here again it is clear that Mill has described an unattainable ideal. It is manifestly impossible to be sure that the instances which are compared "have every circumstance in common save one." For example, they are not in the same place; or, if in the same place, they are not at the same time. All that can reasonably be demanded, therefore, is that the instance of the phenomenon and the instance in which it does not occur shall be as nearly alike as possible. Thus, in the example of the method of difference which was given above, the housewife is said to have cut the two pieces of

butter from the same print. It is therefore probable that the only important difference between them is the circumstance that one is exposed to the heat while the other is not.

Most *experiments* depend upon the method of difference. In an experiment, then, the chief point to be regarded is that the things which are compared shall differ in as few respects as possible. Suppose that a chemist wishes to test a certain liquid to find whether or not it contains arsenic. He may take a portion of the liquid to be tested and an exactly equal portion of distilled water. Each is then put through exactly the same process. That is to say the distilled water—sometimes called the "control"—is heated and stirred and filtered and treated with various reagents just as the other is treated. All the circumstances, then, being the same, if the phenomenon, namely, the reaction for arsenic, is present in the one instance and absent in the other, the chemist is justified in concluding that the cause of this reaction is the presence of arsenic in the original sample. If, however, the experiment is not "controlled," it is always possible that the arsenic revealed by the test might have come from the glass of the test-tubes and beakers, or might have been contained in some of the reagents employed in making the test.

The Method of Concomitant Variations.—Sometimes, however, no instance is available in which the phenomenon is completely absent. In such a case we have recourse to a modification of the method of difference which is known as the "method of concomitant variations." Instead of comparing two instances, in one of which the phenomenon is present and in the other absent, we compare instances in which the phenomenon appears in varying degrees. Mill states the canon of this method as follows:—"Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an

effect of that phenomenon or is connected with it through some fact of causation."

In the example of this method which was given above, one of the two phenomena is the softening of butter, the other is the temperature. As the temperature increases, the butter becomes softer and softer. The same method may be used to prove that heat expands iron. A piece of iron may be heated artificially, and measurements will show that as the temperature increases the length of the iron also increases. Or a rod of iron may be measured every hour of a summer day.

In these examples the phenomena change quantitatively in the same direction. That is to say, as the one increases, the other increases; and as the one decreases, the other decreases. The relation in some cases is, however, inverse rather than direct. In other words, there are phenomena which are so related that an increase in the one is accompanied by a decrease in the other, and *vice versa*. For example, if statistics show that a diminution of the harvest is always accompanied throughout a term of years by an increase of crime, or a decline in sales by an increase in fires, we should in each case conclude by the canon of the method of concomitant variations that there is a causal relation between the phenomena which are compared. In these cases, however, the relation is inverse.

It does not always follow, however, that because two phenomena vary concomitantly one must needs be the cause of the other. For there are cases of this sort where both phenomena are effects of the same cause. In Mill's statement of the canon provision is made for this possibility by the concluding clause, "or is connected with it through some fact of causation." Thus the length of the period of daylight and that of the period of darkness vary concomitantly, since when one increases the other becomes less. Yet neither

is the cause of the other. Both, however, are effects of the same cause, to wit, the motion of the earth relatively to the sun. Another good example is afforded by the discussion of the relation between alcoholism and crime. Prison records show that a very large proportion of those who are convicted of crime are habitual users of alcoholic liquors. From this it has been inferred, perhaps too hastily, that alcohol is one of the causes of crime. There remains, however, the possibility that alcoholism and criminality are not related to each other as cause and effect, but that both are effects of the same congenital weakness.

Errors to Be Avoided in the Use of the Inductive Methods.—Each of these methods has its characteristic fallacy:

1. *Plurality of Points of Agreement.*—As we have seen, it is difficult, if not impossible to find instances of a phenomenon which agree in but *one* circumstance. We are therefore very likely to hit upon some common circumstance, and jump to the conclusion that it is the cause, failing to take into account other common circumstances one of which may be the real cause. This danger diminishes, however, as the number of wisely chosen instances is increased.

2. *Plurality of Points of Difference.*—In the same way, the characteristic error of the method of difference is the failure to take into account the possibility that there may be more than *one* circumstance in which the instance of the phenomenon differs from the instance in which the phenomenon is absent.

3. *Accidental Concomitance.*—It sometimes happens that two phenomena increase or diminish together, when there is no causal relation whatsoever between them. In such cases the lack of real concomitance reveals itself if the investigation is continued for a sufficiently long period of time. Thus in spring or early summer one might be tempted to

infer that there is a causal relation between the increase of the moon and the growth of vegetation; but after a while the moon will decrease again while the corn and the grass keep on growing.

In order to avoid these errors we must never be satisfied with the result indicated by any one experiment or series of observations. The method of agreement should always be supplemented by the method of difference; and even in the employment of the methods of difference and concomitant variations, one should remember that every result is to be regarded as tentative merely,—to be accepted as valid only as long as it has not been shown to be invalid. In general, we may say that the root of all error is undue haste. Indeed, no discovery is accepted by the scientific world as genuine until it has been verified again and again, not only by the original discoverer, but by many other investigators.

The Plurality of Causes.—Thus far we have proceeded as if we could be sure that a given effect is always produced by the same cause. It may be found, however, that instances of the same phenomenon proceed from several causes. Or, at any rate, this is often the result of a superficial analysis. Thus in our account of the hypothetical syllogism we have assumed that a consequent may result from any one of two or more antecedents. For example, suppose that fire consumes a barn filled with new hay. The fire might have had any one of several causes, such as (1) Spontaneous combustion; (2) Lightning; (3) A carelessly discarded match; (4) An accidentally overturned lantern. Accordingly, if a number of barns have been burned, we should not be justified in looking for *the* cause. For there may not have been any *one* cause of all of them; one fire may have been produced by one cause, another by another, and so on. Now where there is such a "plurality of causes," it is obvious that the method of agreement is inapplicable. Since a different

cause is operative in various cases, some circumstance which has nothing at all to do with the causation of the phenomenon might nevertheless be common to a considerable number of instances. If these happened to be the instances examined, the common circumstance would be indicated as the cause of the phenomenon. The likelihood of finding a common circumstance which is not a cause will progressively diminish, however, as the number of instances examined is increased. If the number is sufficiently great, no common circumstance will be found.

We have suggested that the appearance of a plurality of causes may result from an incomplete analysis. For if each of the several causes of the phenomenon is resolved into its elements, some element may be found which is common to all. This ought then to be considered as the cause. Thus, while the burning of the barn might be due, as we have said, to any one of several causes, and one fire might have been produced in one manner, another in another, and so on, the four causes mentioned above are found upon analysis to contain the common element of great heat. And this is, if we speak strictly, the cause. It would be going too far, however, if we were to assert that such a common element always exists. For all we know to the contrary, certain phenomena may be occasioned in two or more ways which are genuinely distinct and irreducible. And even where the existence of a common element is theoretically demonstrable, it may not be possible to isolate this common element in the practical application of the method of agreement. The possibility that there may be a plurality of causes, accordingly, presents a genuine perplexity in the use of the method of agreement. It is another reason for hesitation before accepting the conclusions suggested by this method. Indeed, unless the number of instances is very great, conclusions reached by the method of agreement have little or no value until

they have been verified by the methods of difference and concomitant variations.

Negative Applications of the Methods.—We have seen that a result reached by the method of agreement may be negatived by a further application of the same method, or by the use of the other methods. At the cost of repetition let us make it perfectly clear that the inductive methods may be used, not only to discover and confirm hypotheses concerning causal relations, but also to refute such hypotheses. For example, suppose the phenomenon to be accounted for is the brilliant coloration of certain species of plants and animals. A considerable number of instances of this phenomenon,—that is to say, a great many brightly colored birds, insects, and flowers,—are compared. While differing greatly among themselves, these agree in the circumstance that they are natives of the tropical regions. By the method of agreement the inference is drawn that brilliant coloration is due to the action of light and heat. Suppose, however, that as the investigation is continued it is found possible to assemble an equally numerous group of brightly colored animals and plants which are natives of the temperate zones. These are instances of the phenomenon under investigation, but they do not share the circumstance which was previously supposed to be the cause. Thus the result provisionally attained by a partial employment of the method of agreement is ruled out by a further employment of the same method. Again, if it be found that many desert animals lack bright coloration, the hypothesis which was provisionally adopted is refuted by the method of difference; for instances of the phenomenon and instances of its absence are found to be alike in the circumstance of exposure to light and heat. Finally, if it can be shown that the proportion of colored species does not appreciably diminish as we pass from the torrid to the temperate regions, the hypothesis is refuted by the method

of concomitant variations: for we should expect that the phenomenon of coloration and that of heat and light, if they were causally related, would increase and decrease together.

Induction as the Elimination of Hypotheses.—In reality, then, whether we consider the inductive methods in their negative or in their affirmative applications, they depend upon the elimination of hypotheses. In the method of agreement we vary the instances as much as we can in order thereby to show that the circumstances in which the instances differ are *not* causes of the phenomenon. If the first instance of the phenomenon is attended by the circumstances a, b, c, and no others; and the second instance by a, b, d, e, and no others; we know that c is not the cause of the phenomenon, because we have observed an instance of the phenomenon in which c is absent. If now we find a third instance which has the circumstances b, c, e, and no others, we know that a is not the cause. The cause must then be b, since each of its rivals is absent in at least one instance of the phenomenon. (This account is, of course, very much simplified.)

The method of difference is also a device for the elimination of suggested causes. If the circumstances attending a given phenomenon are a, b, c, d, and no others, and those attending its absence are a, b, c, and no others, it is evident that a, b, and c are ruled out of consideration. In the use of the method of concomitant variations too, a given hypothesis is established by the destruction of its rivals. If A always varies concomitantly with B, while when A is constant C, D, E, *et al.* are changing, and when A is changing some or all of these are constant, it is highly probable that there is a causal relation between A and B, but not between A and C, A and D, etc.

Mr. Joseph (*An Introduction to Logic,* p. 438) has, accordingly, stated the principle of elimination which is pre-

supposed by each of these three methods of induction as follows:

"Nothing is the cause of a phenomenon in the absence of which it nevertheless occurs.

"Nothing is the cause of a phenomenon in the presence of which it nevertheless fails to occur.

"Nothing is the cause of a phenomenon which varies when it is constant, or is constant when it varies, or varies in no proportionate manner with it."

Summary.—The inductive methods are devices to be employed in finding and testing hypotheses. The methods of simple enumeration and agreement lead us to propose various hypotheses for the explanation of a given phenomenon. We then test these proposed hypotheses by the further employment of the method of agreement, or by the methods of difference and concomitant variations. In the methods of enumeration and agreement we compare instances of a phenomenon with other instances of the same phenomenon. In the methods of difference and concomitant variations we compare instances of the phenomenon with instances in which it is absent, or is present in varying degrees.

Exercises.—1. A small boy who was reminded that he had been told to "stop making that noise," replied that he was not making that noise any more, but another just like it. (Bode.) How does the ambiguity in the use of the phrase "that noise" illustrate the distinction between a phenomenon and an instance of a phenomenon?

2. If *beauty* is the phenomenon, what would be an instance of the phenomenon?

3. Specify instances of these phenomena:

(a) Goodness.
(b) Anger.
(c) Death.
(d) Pneumonia.
(e) Germination.
(f) Evaporation.

4. What sort of data would you gather if you wished to prove by the method of agreement that Indian corn is a fat-producing feed?

5. How could the same hypothesis be proved by the method of difference?

6. A man wished to test the agricultural value of lime. He took two plots, applied a coat of lime to the one, and then planted both to corn. The former plot was cultivated five times after the corn was up; the latter was cultivated three times. The former was planted with a yellow variety, the latter with a red variety of corn. The plot which had had the dressing of lime yielded ten bushels per acre more than the other. Criticize this experiment.

7. The teacher of a certain fourth grade compelled all of her thirty pupils to go outside to play at recess, in spite of the fact that snow was falling and the air was cold. Within the next two or three days ten of these children developed severe colds. Some of the parents said that the children had caught cold because they were compelled to play in the snow. Discuss their hypothesis from the standpoint of the methods.

8. Twenty Greek sailors about to put to sea went to the temple of Neptune and prayed to the god for protection. They all returned to their homes in safety. Therefore Neptune had preserved them from the perils of the deep. Criticize the inference.

9. Many people believe that there is a causal relation between inclement weather and a "change of the moon." Show that this belief may have arisen because of a failure to test the result of the method of agreement by means of the method of difference.

10. Which method is illustrated by each of the following? In each case what is the phenomenon, and how many instances are considered? In which do we have an instance of the absence of the phenomenon? In which are the instances chosen so as to be as nearly alike as possible? So as to be as different as possible?

(1) An electric bell was placed under the receiver of an air pump. When the button was pressed the sound was clearly heard. After the air had been pumped out of the receiver (as nearly as possible) the button was pressed, yet no sound was

heard. It was concluded that sound must be transmitted by a material medium.

(2) This hypothesis is also supported by the following modification of the foregoing experiment: The bell was set ringing, and the receiver was gradually exhausted. While the air was being removed, the sound of the ringing gradually died away.

(3) In a certain village there was an outbreak of typhoid fever. Investigation showed that there were cases in well-to-do as well as in poor families. Some of these families had water in their cellars, and some did not. Some bought milk from one dairy, and others from another; while a case appeared in one family that had its own cow. Some families in which the fever appeared had "city water," some had wells, and some depended upon cisterns. It was finally discovered, however, that all who got the fever had attended a picnic in a grove some miles from the village.

(4) The ice on the side-walk melts in the sun. Butter and lard melt in a hot skillet. Cast-iron melts in the great heat of a foundry. Therefore heat is a cause of liquefaction.

(5) Two pieces of iron are taken, of the same weight and shape. The one is allowed to retain its ordinary temperature; the other is subjected to great heat. The former remains rigid; the latter becomes molten.

(6) If we make a careful determination of the quantity of heat applied to a certain mass of iron, and also of the quantity of iron which becomes molten, we discover that there is a definite numerical relation between them.

(7) The more widely he opened the throttle, the faster ran the engine.

(8) Two boys of the same town both went to college. One of them was not interested in college work, and dropped out in the middle of his Freshman year. The other completed his course, and graduated with honors at Commencement. The former became a successful merchant, while the latter received only a small salary as a teacher in a high school. Therefore a college education is not a profitable investment.

(9) A student gave up in despair, because "the harder he worked the more poorly he was able to recite."

(10) If a coin and a feather are dropped at the same instant in the receiver of an air-pump when the receiver is full of air,

the coin falls more quickly. If, however, the air has been removed, they will reach the bottom together.

(11) Sir David Brewster found that various substances, such as beeswax, gumarabic, and balsam, take on the brilliant coloring of mother of pearl if they receive an impress from its surface. The only circumstance in common is the shape of the surface. Therefore the shape is the cause of the color.

(12) Overdriven cattle, if killed before recovery from their fatigue, become rigid and putrefy in a surprisingly short time. A similar fact has been observed in the case of animals hunted to death; cocks killed during or shortly after a fight; and soldiers slain in battle. These various facts agree in no circumstance directly connected with the muscles, except that these have been subjected to exhausting exercise. (Creighton.)

(13) For a long time it had been doubtful whether the red flames seen in total eclipses of the sun belonged to the sun or to the moon; but during the last eclipse of the sun it was noticed that the flames moved with the sun, and were gradually covered and uncovered by the moon at successive instants of the eclipse. No one could doubt thenceforth that they belonged to the sun. (Jevons.)

(14) During the war, a soldier received a bad wound on the head. In the operating room his brain was exposed. All at once the soldier began to talk and continued until it was discovered that part of the skull was resting on the soldier's brain. When the pressure was removed the patient instantly stopped talking. (Snow, *Problems in Psychology*, p. 5.)

(15) Dreams . . . are "wish-fulfilling," to borrow a term from Freud's theory of dreams. . . . A boy dreams repeatedly of finding whole barrels of assorted jackknives, and is bitterly disappointed every time to awake and find the knives gone. . . . An adult frequently dreams of finding money, first a nickel in the dust, and then a quarter close by, and then more and more, till he wakes up and spoils it all. Such dreams are obviously wish-fulfilling, as are also the sex dreams of sexually abstinent persons, or the feasting dreams of starving persons, or the polar explorer's recurring dream of warm, green fields. An eminent psychologist has given a good account of a dream which he had while riding in an overcrowded compartment of a European train, with the window closed and himself wedged in

tightly far from the window. In this uncomfortable situation he dropped asleep and dreamed that he had a seat next to the window, had the window open and was looking out at a beautiful landscape. In all these cases the wish gratified in the dream is one that has been left unsatisfied in the daytime. . . . The newly married couple do not dream of each other. We seldom dream of our regular work, unless for some reason we are disturbed over it. . . .

Any sort of desire or need, left unsatisfied in the day, may motivate a dream. . . . Curiosity may be the motive, as in the case of the individual who, having come to live in Boston, was much interested in its topography, and who saw one day a street car making off in what seemed to him a queer direction, so that he wondered where it could be going and tried unsuccessfully to read its sign. The next night he dreamed of seeing the car near at hand and reading the sign, which, though really consisting of nonsense names, satisfied his curiosity during the dream.

The mastery motive, so prominent in daydreams, can be detected also in many sleep dreams. There are dreams in which we do big things—tell excruciatingly funny jokes, which turn out when recalled the next day to be utterly flat; or improvise the most beautiful music, which we never can recall with any precision, but which probably amounted to nothing; or play the best sort of baseball. The gliding or flying dream, which many people have had, reminds one of the numerous toys and sports in which defiance of gravity is the motive; and certainly it gives you a sense of power and freedom to be able, in a dream, to glide gracefully up a flight of stairs, or step with ease from the street upon the second-story balcony. (Woodworth, *Psychology,* pp. 501f.)

(16) Take a book with a ribbed-cloth binding. Tap the cover with the finger-nail; or pass the nail slowly across two or three ridges. You hear a tapping or plucking or snapping noise. Draw the nail more quickly over a number of ridges. The noise is replaced by a harsh scroop which is distinctly tonal. The pitch of the tonal element rises as the movement becomes quicker. (Titchener, *Experimental Psychology,* vol. I, Part I.)

11. Criticize the following:

(1) A was ill and called in a physician; soon after he died.

B was ill and called in a physician; soon after he died. C was ill and called in a physician; in a few weeks he died. Thus many cases might be enumerated of people who have been ill, have secured the services of a physician, and not long thereafter have passed away. It is therefore dangerous to summon a physician in cases of ill health.

(2) Two men were both victims of dyspepsia. One took Dr. X's patent compound, and was soon well and strong again. The other could not be persuaded to take this medicine, and is still afflicted as before. Therefore Dr. X's compound is a cure for dyspepsia.

(3) According to the booklet of testimonials, a woman in Texas was cured of one disease, a man in Georgia of another, a boy in Maryland of another, and a girl in Ohio of still another disease. As I turn the pages, I discover that many people have been cured of many diseases. I infer that if I take this medicine I shall be cured too.

(4) Most of the generals of the Civil War period wore beards. Therefore beards are conducive to success in the military profession.

(5) Before the adoption of the new tariff law sugar sold at seven cents. Since the new tariff went into effect, the price has risen to twelve cents. Therefore the tariff is responsible for the increase.

(6) The tides are proved to be due to the attraction of the sun and moon, because the periods of high and low, spring and neap tides, succeed each other in intervals corresponding to the apparent revolutions of these bodies round the earth. The fact that the moon revolves upon its own axis in exactly the same period that it revolves round the earth, so that for unknown ages past the same side of the moon has always been turned towards the earth, is a most perfect case of concomitant variations, conclusively proving that the earth's attraction governs the motion of the moon on its own axis. (Jevons.)

(7) If on a calm clear night a sheet or other covering be stretched a foot or two above the earth, so as to screen the ground below from the open sky, dew will be found on the grass around the screen but not beneath it. As the temperature and moistness of the air, and other circumstances are exactly the same, the open sky must be an indispensable antecedent of dew. (Jevons.)

(8) The magnetic compass needle is subject at intervals to very slight but curious movements; and at the same time there are usually natural currents of electricity produced in telegraph wires so as to interfere with the transmission of messages. These disturbances are known as magnetic storms, and are often observed to occur when a fine display of the Northern or Southern Lights is taking place in some part of the earth. Observations during many years have shown that these storms come to their worst at the end of every eleven years . . . and then diminish in intensity until the next period of eleven years has passed. Close observations of the sun during 30 or 40 years have shown that the size and number of the dark spots, which are gigantic storms going on upon the sun's surface, increase and decrease exactly at the same periods of time as the magnetic storms upon the earth's surface. No one can doubt, then that these strange phenomena are connected together, though the mode of connection is quite unknown. (Jevons.)

(9) It was a general belief at St. Kilda that the arrival of a ship gave all the inhabitants colds. Dr. John Campbell took pains to ascertain the fact and to explain it as the effect of effluvia arising from human bodies; it was discovered, however, that the situation of St. Kilda renders a northeast wind indispensably necessary before a ship can make a landing. (Hibben.)

REFERENCES

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Sellars, *Essentials of Logic,* Chap. XV.

For advanced study:

Mill, *A System of Logic,* Book III.
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Schiller, *Formal Logic,* Chapters XVIII and XIX.

CHAPTER X

STATISTICS AND PROBABILITY

1. In a certain school the height of the pupils in inches was found to be as follows: 40, 38, 42, 45, 43, 41, 59, 42, 39, 39, 51, 43, 41, 44, 40, 42, 39, 50, 42, 46, 42, 41, 43, 40, 44, 47, 35, 48, 36, 42, 34.

2. These measurements were obtained in another room: 41, 43, 44, 59, 40, 44, 45, 43, 40, 43, 43, 44, 48, 49, 43.

Which of these groups of children is the taller? Is every child of the one group taller than every child of the other? How then can you say that one group is taller than the other? Imagine each group to be arranged in the order of height. Which child of each group would be in the middle of the line? Which height occurs the most frequently in each group?

Averages.—Except in the rare case that the lowest individual of one group ranks higher in weight, height, virtue, or whatever quality is the subject of the investigation, than the highest of the other group, the comparison of groups has no meaning unless we make use of the idea of averages. We must find some real or imaginary individual which may be taken as typical of the group. The averages which are most frequently employed are the arithmetical average, the median, and the mode.

The *arithmetical average* (known also as the "mean") is the quotient obtained by dividing the sum of all the terms of a given group by the number of terms in the group.

Using the Greek letter Σ to indicate summation, n to stand for the number of terms, and a to represent any term of the group, the arithmetical average is

$$\frac{\Sigma a}{n}$$

The *median* is that member of a group which, when the group is ordered with respect to some measurable quality, stands in the middle of the array. That is to say, there are exactly the same number of terms above the median as below it. If the group has an even number of members, either of the two middle terms may be taken as the median; or, if they differ, their arithmetical average may be taken.

The *mode* is that measure of height, weight, etc., which occurs most frequently. In everyday parlance "mode" is used as a synonym for style or fashion. Thus the mode is the shape of hat or the cut of coat which is most popular or most frequently worn. Suppose that the heights of the men in a certain group are 68, 65, 60, 70, 65, 68, 66, 68, 66, 70, 68, and 72. Inspection reveals that there is one man who is 72, two that are 70, four that are 68, two that are 66, two that are 65, and one that is 60. The height which occurs most frequently is, then, 68, and this, accordingly, is the mode of the group.

Exercises.—In each of the following cases determine the arithmetical average, the median, and the mode:

1. The men employed in one room of a certain factory received weekly pay checks for the following amounts: $12, 20, 32, 28, 18, 21, 20, 16, 20, 24, 15, 20, 18, 21, 16, 26, 20, and 13.

2. The members of a certain club are found to be respectively 50, 25, 62, 45, 22, 45, 48, 30, 45, 36, 24, 50, 45, 48, 25, 30, 45, and 40 years of age.

Normal Distribution.—None of the averages which we have discussed can justly be called the "true" average. Indeed, no average can take the place for all purposes of a detailed knowledge of the group. Which of our averages will represent the group most adequately will depend upon the purpose for which it is used. In practice, however, it is usually found that, given a random selection of items, as the number of items increases, the arithmetical average, the median, and the mode, will approach coincidence. In other words, in nature we find *types* and the number of deviations from the type is inversely proportional to the amount of the deviation. Men of average height are much more numerous than those who are either very tall or very short. There are just as many above the average as below it; and if a "curve" is plotted to represent the distribution, it is symmetrical. The greater the degree of divergence from the average, the fewer individuals there will be who show this deviation.

Several years ago an investigation was made at the University of Illinois Agricultural Experiment Station of the length of the ears of corn grown from seed corn taken from ears of which each was ten inches long.

The following table shows the result of the investigation:

Length of Ears in Inches	Number of Ears at Each Length
3.0	1
3.5	0
4.0	1
4.5	0
5.0	2
5.5	3
6.0	9
6.5	8
7.0	12

In this case the modal length, 9.0 inches, is somewhat greater than the arithmetical average, but it is identical with the median.

(Secrist, *Introduction to Statistical Methods,* p. 211.)

Length of Ears in Inches	Number of Ears at Each Length
7.5	19
8.0	32
8.5	40
9.0	67
9.5	63
10.0	38
10.5	21
11.0	8
11.5	2
12.0	1

The following table of grades given by the Pennsylvania State College in the year 1920-21 also illustrates the tendency toward an approximately normal distribution:

Grades	A	B	C	D	E	Total
Number of each:	4384	19047	14302	2244	768	39745
Percentages	11.03	45.40	35.99	5.65	1.93	100.00

If the distribution were symmetrical the percentage of E's would equal the percentage of A's, and that of D's would equal that of B's. Why this departure from a normal distribution? Two suggestions may be offered: (1) The instructors may be too lenient in their grading; and (2) by reason of entrance requirements and the dropping of unsatisfactory students the student body is a selected group, not including as large a proportion of weaklings as the general population.

The Method of Agreement Applied to Groups.—In our study of the inductive methods, while we did not expressly limit them to the comparison of individual instances, we were chiefly concerned with these. In applying the methods, however, we soon discover that the relation between cause and effect is exceedingly complex. Some hint of this complexity was given in our discussion of the perplexities arising from the appearance of a "plurality of causes." An even more serious perplexity is occasioned by the fact that a "cause" may sometimes be present without producing its

effect. Thus a lighted match put to paper ought to cause a blaze. But sometimes it does not. Perhaps the wind is too strong, or the paper is wet. For some reason or reasons which are not completely understood, radio is still rather uncertain. Sometimes the apparatus works, and sometimes it does not. Of course, all cases in which the "cause" fails to produce its effect, are explained, in principle, by saying that conditions must be right and that there must be no countervailing force. Really the *total* cause is the entire situation; in other words, the state of the universe at the time when the effect occurs. The inductive methods, however, presuppose the possibility of finding some one feature of the situation which may be considered as *the* cause. How, then, can we apply the methods if the cause is active in some cases and not in others?

The answer is that *groups of instances* may be substituted for the individual instances referred to in our previous formulation of the methods. Thus if a certain bacillus is present in the drinking water of a certain village, and we set out to prove by the method of agreement that this bacillus is the cause of typhoid fever, we are confronted by the difficulty that a considerable number of people have drunk the water and yet have not got the disease. Nevertheless, if it could be shown that of various *groups* of people,—such as the well-to-do people of the town itself, the poorer people of the town, summer boarders, automobilists stopping for a few hours, etc.—all of whom drank the water, a *considerable proportion* went down with the disease, we should regard the causal relation as established. These groups, though differing very much one from the other, nevertheless agree in that their members had drunk the polluted water, and that many of them contracted the disease. Therefore, by the canon of the method of agreement, the fever is the effect of the polluted water.

The Method of Difference Applied to Groups.—Even more important is the comparison of groups which depends upon the method of difference. To prove that a certain bacillus is the cause of typhoid, it would not be satisfactory to compare one person who had drunk the water containing it with one person who had drunk water free from it; for the person who had drunk the polluted water might not get sick. But if we should compare two large groups, one composed of people who had drunk the polluted water and the other of persons who had not, and found that a considerable proportion of the former got the disease, and none, or a much smaller proportion of the latter, we should conclude by the canon of the method of difference that there is a causal relation between the ingestion of the bacilli and the appearance of the disease. In the same way we should investigate the relation of vaccination to smallpox or of the anti-toxin treatment to diphtheria. It may happen that some individuals who have been vaccinated will get smallpox; that some patients to whom the anti-toxin has been administered will die of diphtheria. Nevertheless, if a comparison of groups of the vaccinated with the unvaccinated, or of those treated with anti-toxin with those not thus treated, reveals a much higher percentage of cases or of deaths in the former than in the latter, we may justly infer a relation of cause and effect.

The Method of Concomitant Variations Applied to Groups.—Some of the illustrations of concomitant variations given in the previous chapter referred to the variations of groups, or of the averages of groups. Thus we have spoken of the relation of diminishing sales to increasing fires, and of small harvests to agrarian crime. In the same way a connection has been made out, in wine-drinking countries between an abundant grape harvest and certain forms of crime. In these, as in simpler examples of concomitant

variations, the variations take place in *time*. In certain other cases, the variations take place, so to speak, in *space*. Of course, in reading or studying the record we translate space into time. The following argument in favor of vaccination depends upon both the temporal and the spatial correspondence of variations in the completeness of vaccination and in the smallpox mortality:

"In Sweden the population and the smallpox mortality have both been known year by year since 1774. Before vaccination the mortality from smallpox for thirty years averaged 2,045 per million. With permissive vaccination from 1802 to 1816 it was reduced to 480; during seventy-seven years of compulsory vaccination the mortality averaged 155 per million; and for ten years ending 1894 it has been down to 2 per million. . . .

"If we compare the rate of smallpox mortality in the different countries, we see an enormous difference between the well vaccinated and the badly vaccinated populations. Here is a table, given by Dr. Edwardes, of the mortality rates per million in the five years 1889 to 1893:

	Smallpox Mortality per Million
Germany	2.3
England and Wales	13.6
Chief French towns	147.6
Italy	180.8
Belgium	253
Austria	313
Spain	638
Russia, 3 years only, including Asiatic Russia	836

"In Germany, vaccination and revaccination are both compulsory. In the other countries revaccination was, at that time at least, nowhere enforced."

(*Edinburgh Review*, Vol. 189, pp. 350-2, quoted by Bode, *Outline of Logic*, p. 295.)

Correlation.—In the examples just given, while figures are given for the smallpox mortality, no numerical estimate is offered of the completeness or incompleteness of vaccination at different times or in different countries. In such cases no satisfactory mathematical estimate of the closeness of the correspondence between the phenomena is possible. In other cases, however, such as in the comparison of the height and the weight of a group of men, or the length and the width of a group of leaves, definite measurements can be made, and the degree of correspondence can be computed. Where this is possible the correspondence is known technically as "correlation," and the numerical expression as the "coefficient of correlation." If there is a complete correspondence between two qualities or phenomena, the correlation is *perfect* and the coefficient of correlation is 1.[1] If the correspondence is complete but inverse, the coefficient is — 1. If there is no correspondence at all, the coefficient is 0. While if there is a partial positive correspondence the coefficient is a positive proper fraction; and if there is a partial inverse correspondence, the coefficient is a negative proper fraction. In short, the numerical expression for the degree of correlation ranges continuously from + 1, the symbol for a complete positive correlation, through 0, the symbol for the complete absence of correlation, to — 1, the symbol for a complete reversed or negative correlation.

Suppose, for example, that a hundred apple leaves are arrayed, (1) in the order of length, and (2) in the order of width. Would the order be the same in both cases? If it is the same, the correlation is perfect and positive. The first in the one array is the first in the other, the second in the one the second in the other, the third the third, and so on. On the other hand, if the first in the one array is the last

[1] The coefficient of correlation is commonly expressed as a certain number of *hundredths*, as 40, 60, or 80. The coefficient of *perfect* correlation is 100 hundredths or 1.00.

in the other, the second the next to the last, and so on, the correlation is perfect but negative. In either case, if we know the position of a given leaf in one array, we can predict its position in the other. If there is zero correlation, however, a knowledge of the position of a leaf in one array gives no indication whatsoever concerning its position in the other. As a matter of fact, between the length and the width of a considerable number of leaves of the same species, there would undoubtedly be a positive correlation; but it is extremely unlikely that the correlation would be perfect. The value of the coefficient would lie between + 1 and 0.

The Pearson Formula.—Relations of concomitance such as are found between variations in the temperature and in the length of a steel rail, or of a column of mercury, are examples of perfect correlation. Another example is the definite corespondence between the temperature and the pressure of a gas the volume of which remains constant. On the other hand, when the temperature of the gas is constant, the relation of volume to pressure illustrates the idea of a perfect *negative* correlation. In such cases the causal connection between the two phenomena is obvious and indisputable. There are many cases, however, in which the causal relation is masked by the presence of other factors which hinder its operation. In other words, the investigator is perplexed by the occurrence of numerous exceptions. This is especially true in the sciences which deal with the living,—in such sciences as biology, psychology, sociology, economics, and education. In these sciences, perfect correlations, if found at all, occur with considerably less frequency than in physics and chemistry. If the former group are really to be sciences, they must find some method of dealing with correlations which are not perfect. To meet this need, Karl Pearson, one of the leaders in the field of biometrics (i.e. the calculus which deals with the phenomena of the living),

devised a formula for computing the coefficient of correlation which is applicable to all cases. It may be written as follows:

$$r = \frac{\Sigma xy}{n} \div \sigma_1 \sigma_2$$

Here r, of course, is the symbol for the coefficient of correlation. Σ and n have the significance already explained in our account of the arithmetical average. In fact they represent the average of the products of x into y. x stands for the deviation of any item of the first array from the arithmetical average, and y in like manner for the deviation of any member of the second array. σ_1 and σ_2 represent respectively the "standard deviations" of the two arrays. The standard deviation, in each case, is a sort of average of the deviations. More accurately, it is the *square root* of the average of the *squares* of the deviations. Expressed in symbols,

$$\sigma_1 = \sqrt{\frac{\Sigma x^2}{n}} \quad \text{and} \quad \sigma_2 = \sqrt{\frac{\Sigma y^2}{n}}$$

We may, then, think of the coefficient of correlation as the ratio of the average of a series of products to the product of two averages. More accurately, it is the quotient obtained when we divide the average of the products of the corresponding deviations by the product of the standard deviations of the two arrays.

To illustrate the use of the formula, consider the relation between the measures of heights and weights given below. Of course no results of any value could be derived from so small a number of cases. It seems desirable, however, to over-simplify in the interest of a preliminary understanding. Suppose, then, that seven men are measured and weighed with the indicated results. The height, in each case, is given

in inches, the weight in pounds. The average height is 68; the average weight, 140. In the column headed by x, are the deviations from the average height, and under y the deviations from the average weight. As previously explained, these deviations are found by subtracting each measure from the average of its series,—in this example from 68 in the case of the heights, and from 140 in the case of the weights. Thus the first man exceeds the average height by 3 units, and the average weight by 10 units, and so on. Observe that when any item is less than the average of its column, the deviation is negative. The *square* of the deviation, however, is always positive. With these explanations the diagram and the subjoined calculations should be readily understood.

Height	x	x^2	Weight	y	y^2	xy
71	3	9	150	10	100	30
70	2	4	160	20	400	40
69	1	1	155	15	225	15
68	0	0	140	0	0	0
67	−1	1	110	−30	900	30
66	−2	4	135	−5	25	10
65	−3	9	130	−10	100	30
		28			1750	155

$$\sigma_1 = \sqrt{\frac{28}{7}} = \sqrt{4} = 2; \sigma_2 = \sqrt{\frac{1750}{7}} = \sqrt{250} = 15.8; \frac{\Sigma xy}{n} =$$

$$\frac{155}{7} = 22.1. \quad \text{Therefore } r = 22.1 \div 31.6 = .70$$

The Justification of the Formula.—We cannot give a complete mathematical demonstration of the formula. We may, however, commend it to ourselves by observing that it "works." It is meant to produce a value more nearly equal to unity as the closeness of correspondence increases; more nearly approaching 0 as the degree of correspondence di-

minishes; and negative if the correspondence is inverse. That it fulfils these requirements is evidenced by the following considerations:

1. If the correspondence is complete, the corresponding terms of the two arrays are (a) equal, or (b) related by a constant multiplier. If the corresponding terms are equal, their deviations from the average are respectively equal. Accordingly, $x = y$, and $xy = x^2 = y^2$. Consequently the average of the xy's is equal to the square root of the product obtained through the multiplication of the average of the x^2's by the average of the y^2's. $\frac{\Sigma xy}{n} = \sigma_1 \sigma_2$. Therefore $r = 1$.

If one of each pair of terms is a constant multiple of the other, the situation is really the same as that just described; for it is obvious that Case (b) could be transformed into Case (a) by a different choice of units, as some fraction or multiple of a foot instead of a foot in the measurement of height, and some fraction or multiple of a pound instead of a pound in the measurement of weight. For example, if the weight in pounds should in each case be twice the height in feet, numerical equality could be obtained by making the unit of height *six inches,* or the unit of weight *two* pounds.

2. However much the deviations of each pair may differ numerically, if they are always in the same direction, or always in different directions, the correlation, whether positive or negative, is fairly high. For all the xy's are positive, and when added none offsets the others.

3. If in some pairs, however, the deviations are in different directions and in others in the same direction, the correlation, whether positive or negative, is low. For the xy's will be of different signs and when added will offset one another.

(a) If positive and negative xy's balance, i.e. if their algebraic sum is 0, $r = 0$.

(b) If positive xy's overbalance negative xy's, r will fall above o.

(c) If negative xy's overbalance, r will fall above o.

The Quartile Measure of Correlation.—When a series of measures is arrayed in the order of magnitude, the middle term of the array, as we have seen, is called the *median.* It is obvious that each of the semi-arrays may be divided in the same manner. The term which thus subdivides one of the halves of an array is the *quartile.* The middle term of the upper half is the upper quartile; that of the lower half is the lower quartile. The median and the two quartiles thus divide the whole array into four parts. If the data to be considered are numerous, as they must be if the result of the calculation is to be of any value, considerable labor is required to derive the coefficient of correlation by the Pearson formula. The degree of correlation may, however, be estimated with sufficient accuracy for most purposes by the use of the quartile-division. Let each of the groups which are to be compared be arrayed and divided by the median and quartiles. Then observe what proportion of the terms of each of the four divisions of the first array are found in the corresponding division of the second array. Thus if the degree of correlation is high (and positive), a large proportion of those which are above the upper quartile of the first array will also be above the upper quartile of the second; if the degree of correlation is high (but negative), many which are above the upper quartile of the first will be below the lower quartile of the second; if there is only a small amount of correlation, those found in the highest division of the first will be scattered among all the divisions of the second, with, perhaps, a somewhat greater proportion of them in the highest (if the correlation is positive) or in the lowest (if the correlation is negative); while if there is no correlation at all between the two arrays, the terms which

are in the highest division of the first will be uniformly distributed among the four divisions of the other.[1]

Exercises.—1. Let the following be the grades given to the pupils of a certain school in English, History, and Arithmetic. Calculate the correlation, (a) between English and History; (b) between English and Arithmetic; and (c) between History and Arithmetic.

Pupils	English	History	Arithmetic
T. A.	80	90	95
E. B.	100	90	90
J. B.	70	70	70
R. D.	75	70	80
M. E.	95	75	80
A. F.	85	70	100
H. F.	90	90	90
N. F.	85	80	90
B. G.	70	75	80
C. H.	85	90	70
J. L.	100	100	100
R. M.	90	85	85
D. N.	85	90	85
S. N.	80	85	75
B. P.	75	90	80
J. R.	95	95	100
W. R.	80	85	85
H. S.	90	95	80
M. S.	85	80	70
P. S.	90	90	95
T. S.	80	85	95

Husband	Wife
22	18
24	20
26	20
26	24
27	22
27	24
28	27
28	24
29	21
30	25
30	29
30	32
31	27
32	27
33	30
34	27
35	30
35	31
36	30
37	32

[1] It is manifest that if there is no correlation at all between the two arrays, about 25 per cent of the terms should be in the same quartile-division. If, then, we have found the percentage of agreements, the result may be translated into an expression roughly corresponding to the Pearson value for r, by the formula, $r = \frac{p - .25}{.75}$, where p is the percentage of correspondence found by the quartile method.

2. Calculate the coefficient of correlation for ages of husband and wife according to the data in the above table. (See King, *Elements of Statistical Method,* p. 201.)

Correlation and Probability.—In calculating correlations we obtain results which may be carried out to two or three places to the right of the decimal point. But let us beware of the *illusion of accuracy!* The accuracy with which results may be expressed is no ground for supposing that these results have any very precise signification. After all, the coefficient of correlation is only an estimate of probability, and a few points more or less do not make much difference. It is an estimate of the likelihood that an individual that stands high with respect to one quality will be found to stand high with respect to another. Correlations of less than .30 had better be disregarded; the correspondence revealed by them is too slight to have much meaning. On the other hand, a correlation of .70, .80, .90, or more gives a likelihood approaching certainty that there is some causal relation connecting the qualities or phenomena in question. The theory of correlation leads, then, to a discussion of probability.

The Constancy of Classes.—We may begin our account of the theory of probability with the proposition that it is easier to formulate laws which hold true of classes than to formulate laws which hold true of individuals. Paradoxical as the statement may appear, we know more about large collections of things than we do about their individual members. Indeed, it has been suggested that the absolute reign of law in the physical world is an illusory appearance resulting from our ignorance of the idiosyncrasies of the individuals which compose the physical world. On this view the laws of physics have merely a statistical validity. They simply describe the *average* behavior of a great number of diverse units, each of which has an individuality of its own. We

are not personally acquainted, so to speak, with molecules, atoms, and electrons; we know them only in multitudes so numerous that individual peculiarities offset each other. It is not inconceivable that the molecules of mercury, let us say, are just as individual, not to say "temperamental," as human beings. For all we know to the contrary, they may differ, for example, in their capacity for heat as much as school boys in their capacity for knowledge. Nevertheless mercury, in the quantities with which we deal with it, has a definite "specific heat," the knowledge of which enables us to predict just how much heat will be required to increase the temperature of a given quantity of mercury by any assigned number of degrees.

Whether this is the true view of the significance of laws in physics and chemistry may well be doubted. We have referred to it here only to illustrate the fact that knowledge of the mass may coexist with ignorance concerning the individual. Individuals vary; the class remains constant. Events such as deaths, marriages, divorces, and suicide seem to be quite individual and unpredictable. Nevertheless "vital statistics" remain surprisingly constant from year to year. In a given population, there are each year about the same number of births, deaths, marriages, etc. The same is true of events such as fires, automobile accidents, etc. It is true of the letters sent annually to the Dead Letter Office. And, in general, if the class of cases is sufficiently numerous, prediction within reasonable limits is always possible.

Insurance.—The principle of the constancy of classes has many practical applications. One of the most instructive of these is insurance. How can an insurance company afford to insure a man against death, fire, or accident? The purchase of a policy certainly does not prevent any of these misfortunes. Furthermore the company does not know whether or not Mr. Smith will die, or his barn burn, or some

member of his family fall and break a bone. The actuaries, however, know that, in a given time, approximately a certain number of men of Mr. Smith's age and apparent state of health will die; that approximately a certain number of barns will burn and bones be broken. In other words, while the insurance company has no knowledge whatsoever of what will befall the individual, it bases its operations upon its knowledge of the class to which the individual belongs.

The Mathematical Expression of Probability.—We have just said that even when we are ignorant concerning the fortunes of the individual we may rely upon our knowledge of the class to which the individual belongs. *A statement concerning a class may, however, take the form of a statement concerning an individual.* Concerning the individual, it is a statement of *probability*. Thus we say that *it is probable* that a man of 25 will live to be 30. This is, in form, a statement concerning an individual; but it really refers to a class. It means that most of the men of 25 will be alive five years hence. If statistics show that out of a million men of a given age a thousand will die within the year, the probability that X, who is of this age, will die within the year is 1/1000. (It is assumed that we know nothing about X except his age.) In general, the probability of any event is represented mathematically by a fraction the numerator of which is the number of cases which are "favorable," i.e. which comply with the requirements of the problem, while the denominator is the number of cases which, in the light of our knowledge when the estimate is made, are *possible*. To take the most obvious example, when we toss a coin the probability of "throwing heads" is ½, because there are two possibilities, and one of these is favorable. Again, suppose that one side of a cube is green, two sides are blue, and three sides red. What is the probability that the cube will fall with a given color uppermost?

It is obvious that there are six possible positions of the cube; for we can give no reason for expecting one of these rather than another. Now of these six, one is favorable to green, two to blue, and three to red. Accordingly, by definition, the probabilities are respectively 1/6, 1/3, and 1/2. It follows from the conventions which have been adopted that the greater the fraction, the higher the degree of probability; the smaller the fraction the less the probability. When the numerator is the same as the denominator, all the possible cases are favorable cases. The event is sure to occur. Therefore 1 is the symbol for certainty. When, however, the numerator is 0, none of the possible cases is favorable. The event is impossible. Therefore 0 is the symbol for impossibility.[1]

The Relativity of Probability.—We must be on our guard, however, against a not uncommon misunderstanding. One might suppose that probability implies contingency or indetermination. But this is not the case. On the contrary, *an event which is merely probable may be absolutely necessary.* In other words, probability has nothing at all to do with the causation of an event, but merely with our knowledge or ignorance of the data upon which we must rely in predicting whether or not it will occur. For example, the probability that a youth of twenty will be alive ten years hence is greater than that of either a babe of six months or a man of sixty summers. Yet, as a matter of fact, the youth may die to-morrow. But, even, if he should, the statement of probability would not be invalidated. An omniscient intelligence would, indeed, have fore-known his death; but the estimate of relative probability was justified by all that was humanly known at the time the judgment was made. It

[1] Observe that the value of r, the coefficient of correlation, ranges from + 1 to — 1, while the numerical expression of probability ranges only from 1 to 0. A correlation of — 1, therefore, corresponds to a zero probability, and a correlation of 0 to a probability of ½. These equivalences should be borne in mind in interpreting our statement that correlation is an estimate of probability.

was probable that he would outlive the others. An event may be more probable at one time than at another; that is, with respect to one body of information than with respect to another. Thus, to return to the actuaries' computation of the expectancy of life, the probability that a man of a given age will be alive a year hence is, let us say, 9/10. This is the probability if we know nothing about him except his age. If, however, we learn that this particular man has heart trouble, the probability in the light of our greater knowledge becomes less than it was before.

Again probability may increase with added knowledge. Suppose that a week before the election has taken place the probability that a certain one of the candidates will be inaugurated as the President of the United States is 3/5; after the election, if this candidate has been successful, the probability will be much greater. If, then, P stands for *probability, a* for a certain event, and m/n for the mathematical estimate of the probability of this event, we should write the formula, not simply, $P\ a = m/n$, but more accurately,

$$P_d\ a = m/n$$

Here the subscript *d* stands for the *data,* that is to say, the body of information in the light of which the estimate of probability is made. Even when the subscript is not written, it should nevertheless be understood that the estimate is relative to a certain body of knowledge.

The Calculation of Probabilities.—For the most part our judgments of probability are vague statements of our "feeling" that one event is more likely to occur than another. Many of them, indeed, do not rise above the level of mere unanalyzed presentiments or forebodings. Mathematicians, however, as we have seen, define probability as the ratio of all the cases which are known to be favorable to all the cases

which are known to be possible.[1] If it be granted, then, that the probabilities of the separate events are known and may be expressed in the manner indicated, the probability of a compound event may be calculated. We shall consider, first, the more obvious cases of the compounding of events, and then take up a few of the complications. By the *combined occurrence* of two events we shall mean the occurrence of both events, either simultaneously or one after the other. By the *alternative occurrence* of two events we shall mean the occurrence of one *or* the other of them. The former is the *logical product* of the events; the latter is their *logical sum.* For example, if the rules of an organization provide that no meeting shall be held unless either the president or the vice-president be in the chair, it would be pertinent to inquire as to the probability that *both* will be present, and even more pertinent to inquire as to the probability that at least one of these officers will be present. It is manifest that the combined occurrence of two events is usually *less* probable than that of either event separately; while the alternative occurrence of events is usually *more* probable than the separate occurrence of either of them.

The Combined Occurrence of Two Events.—Suppose that the probability of each event is known, how can we find the probability that both will occur (either at once or one after the other)? The rule given by the mathematical students of probability is that the probability of a compound event of this sort is equal to the *product* of the separate probabilities of its components. For example, if one tosses a coin the probability of throwing heads is ½. If, now, two coins are tossed at once, or one after the other, the prob-

[1] It is obvious that this definition presupposes the *equi*-probability of the possible cases. At first sight this may seem to be an unwarranted assumption; for it is hard to see how one could prove that one possible case is no more and no less probable than another. But if probability is *relative to the data at hand,* equi-probability means no more than the *absence* of any reason for expecting the occurrence of one rather than another.

ability in the case of each is still ½, inasmuch as neither coin affects the other. The probability that both will be heads (or that we shall throw heads twice in succession with the same coin) is then only ½ of ½, or ¼. Or, suppose that we paint two cubes so that in the case of the one two of the surfaces are red and the remainder blue, and in the case of the other one is red and the remainder blue. If both are tossed the chance of falling with a red horizontal surface uppermost will be 1/3 in the case of the former, and 1/6 in the case of the latter. And, by the rule for combined occurrence, the chance that red will be uppermost in both cases is $1/3 \times 1/6 = 1/18$.

The result may also be obtained by analysis, and the generalization of this analysis is the proof of the rule. When two coins are thrown (or the same coin twice), there are evidently four possibilities, which so far as our knowledge goes are equally probable: (1) both heads; (2) one head and one tail; (3) one tail and one head; (4) both tails. The requirement is that both be heads; hence only one of the four possibilities is "favorable." In the case of the cubes, since any of the six surfaces of one may be combined with any of the six surfaces of the other, there are 36 possible combinations. The requirement is that red of one shall be combined with red of the other; but, as only two sides of the first cube are red, and only one of the second, there are only two ways in which the required combination may be effected. Hence only two of the thirty-six possibilities are "favorable."

The Alternative Occurrence of Two Events.—In this case the problem is satisfied with the occurrence of either of the two events. Suppose that the probability of their separate occurrence is known, what is the probability that one or the other of the two events will occur? The rule given by the mathematicians is that the alternative prob-

ability of two events is equal to the *sum* of their separate probabilities. For example, suppose that the faces of a cube are painted so that one is green, two are blue, and three red. What is the probability that the upper horizontal surface will be *either* green or blue? There are six possible positions of the cube. Of these, one is favorable to green, two to blue, and three to red. Consequently three are favorable to green *or* blue. The probability of green is therefore 1/6, that of blue is 1/3; while that of green *or* blue is $1/6 + 1/3 = 1/2$.

Exercises.—If it is known that an urn contains two red balls, one white ball, and two black balls, and that another urn contains one red ball, two white balls, and three black balls, what is the probability of drawing red from both urns, if we draw only one ball from each? Under the same conditions, what is the probability of drawing black from both?

2. The conditions remaining the same, what is the probability of drawing red *or* white from the first urn? Of drawing black *or* white from the second?

3. An urn contains four white balls and five black balls. What is the probability of obtaining white at the *second* drawing (but one ball being taken at a time), (a) if the first drawing was black; and (b) if the first drawing produced a white ball which was not returned to the urn?

4. Suppose the urn to contain the same contents as in No. 3. What is the probability of obtaining white at both the first and the second drawing? (When one of the two events depends upon the other, the rule for finding the probability of their combined occurrence must be qualified. Observe that the removal of a white ball at the first drawing diminishes the likelihood of getting white the next time. We should, accordingly multiply the probability of the first event, not by the independent probability of the second, but by the probability of the latter on the assumption that the former event has already occurred.)

5. Given two cubes, one having two red faces and the rest blue; and the other, two blue faces and the rest red. If both are dropped at random upon the table, what is the chance that at least one of them will have a blue face uppermost? (We are

tempted to say that the probability must be $4/6 + 2/6 = 1$. But it is not certain that a blue face will be uppermost. The difficulty arises from the fact that the events are not mutually exclusive. Both may occur. Accordingly each of the separate probabilities, as it were, overlaps the other by a value equal to the probability of the combined occurrence of the two events. Since this has been counted twice, it should be deducted from the sum of the probabilities.)

Complications.—The exercises and accompanying suggestions have directed attention to the qualifications which must be added to the rules for compounding probabilities. These rules may now be stated as follows:

1. To find the probability of the combined occurrence of two events,

(a) When neither event is dependent upon the other: multiply the probability of the first by the probability of the second. $P_d\, ab = P_d\, a \;.\; P_d\, b.$

(b) When one event is dependent upon the other: Multiply the probability of one event by the probability of the other *on the assumption that the former has occurred.* $P_d\, ab = P_d\, a \;.\; P_{ad}\, b = P_d\, b \;.\; P_{bd}\, a.$

2. To find the alternative occurrence of two events,

(a) When the events are mutually exclusive: Add the separate probabilities.

$$P_d\,(a + b) = P_d\, a + P_d\, b.$$

(b) When neither event excludes the other: Add the separate probabilities and from this sum subtract the probability of the combined occurrence of the two events.

$$P_d\,(a + b) = P_d\, a + P_d\, b - P_d\, ab.$$

Under each rule it is obvious that the second case is the more general, and that the first is a special application of it. Thus 1 (a) is included by 1 (b); for if neither event in any way depends upon the other, $P_d\, b = P_{ad}\, b$ and $P_d\, a = P_{bd}\, a.$

Consequently the second formula reduces to the first. In the same way 2 (a) is a special case of 2 (b); for when the events are mutually exclusive, $P_d\, ab = 0$, and here again the second formula reduces to the first.

While the formulas as stated apply to but two events, it is manifest that they may be extended to cover the compounding of any number of events.

The Probability of Non-Occurrence.—In the calculus of probabilities we may apply some of the principles of Symbolic Logic. Let a′ represent the non-occurrence of a, b′ the non-occurrence of b, etc. Then by the formulas of DeMorgan,[1] as also by common sense,

$$(ab)' = a' + b' \text{ and } (a + b)' = a'b'$$

Now it is manifest that $P_d\, a + P_d\, a' = 1$. Hence $P_d\, a = 1 - P_d\, a'$. Also, since $(a + b)' = a'b'$, $(a + b) = (a'b')'$. Hence $P_d\, (a + b) = P_d\, (a'b')'$. These results give us another method of calculating the probability of alternative occurrence. For the probability that at least one of two events will occur is equal to the probability that the combined non-occurrence of the two events will not happen. For example, if $P_d\, a = 2/3$ and $P_d\, b = 3/4$ $P_d\, a' = 1/3$ and $P_d\, b' = 1/4$. Therefore $P_d\, ab = 1/3 \cdot 1/4 = 1/12$. And $P_d\, (a'b')' = 1 - 1/12 = 11/12$. This represents the chance that it will *not* be true that both will fail to occur.

Exercises.—1. A door is fastened with two locks. If the chance that I shall be able to guess the combination of the first is 1/6 and of the second 1/8, what is my chance of being able to open the door?

2. If my chance of hitting the target at the first trial is 1/5, and of hitting it at the second trial (provided that I was successful the first time, and thus encouraged) is 1/4; what is the probability of hitting the target at both trials?

3. A family consists of the parents, one son, and one daugh-

[1] See page 133.

ter. The probability that each will be alive ten years hence is as follows: the father, 9/10; the mother, 7/8; the son, 11/12; the daughter, 15/16. What is the probability that in the course of the decade—

(a) The father will die?
(b) The mother will die?
(c) Both parents will die?
(d) A parent will die?
(e) The whole family will die?
(f) There will be a death in the family?

Probability and Deductive Inference.—In our theory of probability we may substitute the truth of propositions for the occurrence of events. Inference, both deductive and inductive, may thus be discussed from the standpoint of probability. In general, if *c* be a *conclusion,* and *d* represent the premise or premises by which it is supported, we may write, in the case of any recognized "mood" of deductive inference, $P_d c = 1$. In other words, if the premise or premises be granted, the conclusion is certain. For example, let *a* be a categorical proposition of type E, and let *c* be its simple converse. Then $P_a c = 1$, for E may be converted simply. Again, let *a* be the major premise, and *b* the minor premise of a syllogism. Then $P_{ab} c = 1$, if the syllogism is valid.

The moods which deductive logic recognizes as valid are, then, those in which the conclusion follows *certainly* from the premises. It is important to observe, however, that some of the so-called invalid moods are not wholly without evidential value. While they do not establish the conclusion, they do nevertheless yield a greater or less degree of probability. Let us consider but two examples. (a) Take first the case of simple conversion of the A proposition. This we have pronounced fallacious. And yet, if we know that *All A is B,* while we are not *sure* that a particular member

of the class B in which we happen to be interested is also a member of the class A, we have more right to expect it to be than we should have if we knew *nothing at all* about the relation of the two classes. Indeed, if *All A is B,* it is rather probable that *All B is A.* If, then, *a* represents the original proposition, and *c* its simple converse, the evidential value of the inference may be expressed, $P_a c = m/n$, where m is less than n. (b) Consider next an invalid mood of the categorical syllogism in which the attempt is made to draw a conclusion from two particular premises. Though the argument is invalid, it still has some evidential value. This is expressed by the equation, $P_{ab} c = m/n$. If *Some A is B* and *Some C is A,* it is more or less probable that *Some C* is *B.*

Probability and Induction.—While deductive inferences are not ordinarily regarded as valid unless the probability of the conclusion is 1, *induction* never gives more than a *preponderance of probability* in favor of the hypothesis which is said to be established. The only exception to this statement is afforded by the doubtful case of "perfect induction," which, as we have seen, is scarcely to be considered a case of inference at all. The general expression for inductive inference may then be $P_d c = m/n$, where *d* represents the relevant facts and *c* the hypothesis which is adopted to "explain" them. The proposition, *All men are mortal,* may be said to have been established inductively. Yet, even if we suppose it to be true of all men now living, we cannot, in the nature of the case, examine all the instances which would have to be considered to give absolute certainty. Some time in the future men *may* be born who will not die. Thus the future must always evade consideration, and consequently, for this reason, if for no other, probability must always fall short of certainty.

There are two implications. On the one hand, *we have no right to demand absolute certainty* in the case of induc-

tive inference. *A preponderance of probability* is all that we have a right to ask for in the verification of the hypotheses of natural science. A hypothesis has been *proved* when it has been shown to be more probable than any of its rivals.

On the other hand, *scientific conclusions are always tentative.* Fuller knowledge may lead to the acceptance of other hypotheses. And if, with the progress of science, one hypothesis is thus replaced by another, it does not follow that the discarded hypothesis was erroneous, or that those who held it were mistaken. Probability, we have seen, is always relative to a body of knowledge. We represent it, not simply by P, but by P_a. *It is probable* that a youth of twenty will outlive his grandfather. The statement concerning probability is true, and always will be true, even if the youth die first. It is true, and always will be true, *with respect to a certain group of data.* In the same way a hypothesis which, with respect to the data now available, has a preponderance of probability in its favor, may nevertheless, in the light of greater knowledge, give way to some other hypothesis.

Exercises.—1. What size of men's collars is the mode?

2. Can you reconcile the following sets of statistics:[1]

(a) "The report of the collective investigation committee of the British Medical Association, on the subject of 'Temperance and Health,' and the results embodied in it, are both interesting and important.

"A schedule of inquiries was forwarded to all members of the British Medical Association, one hundred and seventy-eight of whom responded, and gave in the aggregate particulars regarding four thousand two hundred and thirty-four cases of deceased lives, aged twenty-five and upward, in which the alcoholic habits of the lives were recorded. For the purpose of investigation, the habits of the deceased with reference to alcohol were divided into five classes, namely: (a) total abstainers; (b) habitually temperate; (c) careless drinkers;

[1] See Bode, *Outline of Logic,* p. 300.

(d) free drinkers; (e) decidedly intemperate. The ages at death of those in each class were registered, together with the causes of death; and the average for each class is given in the following schedule:

Total abstainers	51.22	years
Habitually temperate drinkers	63.13	"
Careless drinkers	59.67	"
Free drinkers	57.59	"
Decidedly intemperate drinkers	53.03	"

(Quoted in the *Arena,* Vol. 8, pp. 209-210.)

(b) "As to the relative heathfulness of temperance or drink the tables yearly made up by the United Kingdom Temperance and General Provident Institution for Mutual Life Insurance (established 1840) afford conclusive practical evidence. The secretary of the institution, Mr. Thomas Cash, kindly furnished me with the following condensed but lucid statement:

	Temperance Section		General Section	
	Expected Claims	Actual Claims	Expected Claims	Actual Claims
1866-70 (five years)...	549	411	1008	944
1871-75 (five years)...	723	511	1268	1330
1876-80 (five years)...	933	651	1485	1480
1881-82 (two years)...	439	288	647	585
Total (17 years)...	2644	1861	4408	4339

"It will be seen from this that the claims in the temperance section are only a little over seventy percent. of the expectancy, while in the general section they are but slightly below expectancy." (Axel Gustafson, *The Foundation of Death,* p. 268.)

3. A new janitor has a bunch of twenty-eight nearly similar keys, one for each door of the building. What is the probability of his being able to unlock the first three doors with only one trial each? Solve also on the supposition that he marks the keys as he discovers them. (Weld, *Theory of Errors and Least Squares,* p. 47.)

4. What is the probability that a whole number of four figures, selected at random, will have two figures alike, and the other two figures alike? (*Ibid.*)

5. One purse contains five sovereigns and four shillings; another contains five sovereigns and three shillings. One purse is taken at random and a coin drawn out. What is the chance that it be a sovereign? (Whitworth, *Choice and Chance,* Fifth Edition, 148.)

6. What would have been the chance of drawing a sovereign if all the coins had been in one bag? (*Ibid.*)

7. Find the correlation between the following sets of "index numbers":

Year	Prices	Unemployment
1912	110	70
1913	100	120
1914	90	140
1915	90	100
1916	110	70

(Davies, *Introduction to Economic Statistics,* p. 149.)

8. Find the coefficient of the correlation for the indexes of normal seasonal variations of merchandise exports from the United States and the price of sterling exchange at New York, as follows:

Month	Exports	Sterling
January	110	100
February	95	108
March	99	109
April	90	115
May	87	116
June	80	120
July	78	119
August	85	106
September	98	74
October	125	70
November	123	80
December	130	83

(*Ibid.*)

9. Find the coefficient of correlation between the following indexes of seasonal variation in the visible supply and price of wheat in the United States, based on the years 1909-1913.

Month	Visible Supply	Price (first of mo.)
January	139	97
February	130	100
March	122	101
April	112	101
May	89	103
June	69	106
July	52	104
August	60	100
September	77	97
October	99	97
November	118	98
December	133	96

(*Ibid.*)

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CHAPTER XI

ANALOGY AND CLASSIFICATION

The Meaning of Analogy.—"Analogy," says Mr. Joseph, "meant originally identity of relation. Four terms, when the first stands to the second as the third stands to the fourth, were said to be analogous, or to exhibit an analogy." (Introduction to Logic, p. 532.) Thus "it might be said that the relation of his patients to a doctor is the same as that of his customers to a tradesman."

For our present purpose, however, it will be sufficiently precise to define analogy as *inference which is based upon resemblance.* Two things are known to be alike in certain respects, and we infer that they will be found to be alike in certain other respects.

The Risk of Error in Analogical Reasoning.—We have just said that, given two things which are alike in certain respects, we infer that they will be found to be alike in certain other respects. Suppose, however, that a tall man with red hair, blue eyes, and a long nose is found to be a thief. Does it follow that every man who is tall and has red hair, blue eyes, and a long nose is a thief? Manifestly not; for resemblances such as these are superficial and unimportant when it is a man's moral character which is the subject of investigation. And, side by side with these points of resemblance there may be, and in all probability are, points of difference which are much more significant.

An argument which takes account of the resemblances of two or more things while ignoring important points of dif-

ference, is said to be based upon a "false analogy." Thus the wise men of the Middle Ages employed a false analogy when they argued against the hypothesis that there were human beings on the other side of the terrestrial globe. They said that people would *fall off* the *under side* of the earth, just as unattached objects fall off the under side of an ordinary ball or sphere. They based their reasoning upon the obvious resemblance between the earth and a small sphere, but overlooked the difference in size between the earth and such a sphere. For an ordinary sphere or ball is a very small body, which is very close to a very large body (the earth); while the earth is not correspondingly near to any body larger than itself. Or, to put the point in a different way, while unattached objects fall off the under side of an ordinary globe, this is impossible in the case of the earth, for the reason that the earth has no *under* side.

Argument from analogy is therefore never safe unless the resemblances upon which it is based are numerous and important, and unless there are no important differences between the things which are compared; while the more numerous and important the points of resemblance, and the less numerous and important the points of difference, the greater the degree of probability that the inference is correct.

Some Examples of the Naïve Use of Analogy.—1. Because the earth and Mars agree in the common possession of a solid crust, an atmosphere, presence of water, changes of season, the possibilities of rain and snow, and other observed qualities, they will also agree in the further respect of being inhabited.

2. The Zulu chews a bit of wood to soften the heart of the man he wishes to buy oxen from, or of the woman he is wooing. (Tylor.)

3. The Illinois Indians made figures of those whose days they desired to shorten, and stabbed these images to the heart. (Dorman.)

4. The native of Borneo made a wax figure of his enemy in the belief that as the image melted, the enemy's body would waste away. (Tylor.)

Cases in Which Analogy Is Dependable.—Between inferences such as those given above and instances of reasoning in which the argument from analogy is justified, there can be no sharp line of distinction. The difference between "false analogy" and "true analogy" is a difference of degree merely. As more and more points of resemblance are discovered, in the absence of important points of difference, we more and more confidently expect the discovery of other points of resemblance. Let A and B resemble one another in the common possession of the qualities *x, y, z,* etc. Suppose, further, that A is known to possess the quality *m.* Then, reasoning from analogy, we should expect to find the quality *m* in B also. If subsequent observation confirms this expectation, we shall predict with even greater confidence that a quality afterwards discovered in A will also be found in B. As we shall presently discover, the argument from analogy, when it is sound, merges into the argument from common membership in a class. For classes of objects, such as horses, men, cities, are set apart and given names on the ground of the common "nature," that is to say, *the complex of qualities possessed in common* by the objects which belong to them. Thus we argue with complete confidence, "If this animal is a dog, it will eat meat"; "If that is a cow, she has four legs"; "If this is a man, he is mortal."

The Probability of Analogical Inference.—There is a temptation at this point to attempt a calculation of the *probability* of specific cases of analogical reasoning. If two things are alike in seven respects and different in three, there is apparently a probability of 7/10 that an additional property or quality of one will also be found in the other. Such calculations, however, have very little value; for not only

the number but also the importance of the points of agreement and of difference ought to be taken into account.

Analogy and Induction.—By some writers analogy has been considered a third type of reasoning co-ordinate with deduction and induction, because in analogical inference we proceed, neither from the general to the particular, nor from the particular to the general, but from one particular to another particular. It is, however, a more defensible position to maintain that analogy, that is to say, inference founded upon resemblance, is the basic type of reasoning, of which induction and deduction are varieties.

Induction obviously depends upon resemblance, since it is only on account of the elements of sameness in particular events that we are justified in thinking of them as cases or "instances" of the same "phenomenon." Suppose that we are inquiring into the causation of an epidemic,—influenza, let us say, or typhoid fever. We seek for the circumstance which is present in every case of the disease and never present when the disease is absent, or for some phenomenon the increase or diminution of which is uniformly correlated with the increase or the diminution of the disease. But this inductive search is impossible unless we are able to diagnose the disease in each case as fever or influenza, as the case may be. In order that the cases studied may be recognized as cases of the same disease, certain "symptoms" must be uniformly present. In short, a great deal of preliminary work must be done before it is possible to apply the inductive methods; and this preliminary work—the classifying of diverse events as instances of a phenomenon—is of the nature of analogy.

Analogy and Deduction.—It is equally clear that analogy is at the bottom of deductive reasoning. Consider the syllogism, "If this is a man, he is rational (or, All men are rational); John Smith is a man; Therefore John Smith is

rational." The minor premise asserts that a certain organism which we call "John Smith," belongs to the class "man." But how can we know this? Obviously because the individual in question resembles other individuals, whom we have already agreed to call "men." In other words, the syllogism concludes that John Smith is rational on the ground that, since he resembles certain other individuals in the attributes which together compose the essence known as *humanity,* it is highly probable that he is also like them in the possession of the attribute of rationality.

The Dependence of Classification upon Analogy.—All inference which depends upon the membership of an individual or of individuals in a class, as well as that in which we pass from one individual to another, depends upon analogy. For a class is but an analogy which is more than ordinarily "complete." Because there are so many points of resemblance among certain individuals that we confidently expect them to be alike in many other respects, it is convenient to give all such individuals a common name, as *man, horse, city, virtue.* Where the resemblance is less complete, or where it is newly discovered, class names are usually wanting; and we usually restrict the notion of reasoning from analogy to these. But it is important to bear in mind that the difference between these cases, and those in which the individuals have been given a class name on account of manifold likeness, is only a difference of degree; and it is therefore profitable to subsume *class* under *analogy,* and to treat the latter as the more general notion.

It must not be supposed, however, that the analogy is ever wholly complete even between members of the same class.[1] For the members of the same class—as two men or two

[1] An absolutely complete analogy would be a case of *identity.* This principle the philosophers have named the *"identity of indiscernibles."* Two things which are alike in all respects,—including position in space and time,—are not two but one.

horses—are never simultaneously in the same place, and more often than not, they are at different times, of different sizes, colors, etc. Nevertheless inference founded upon common membership in such "natural kinds" or classes is the ideal which analogy must approximate in order to lay claim to validity.

Resemblances as Indices of Further Resemblance.—Before going further with our discussion of classification, it may be well to turn aside to inquire concerning the rational basis of analogy. Why do we expect further resemblance because certain resemblances have been discovered? In a subsequent chapter[1] we shall examine the problem more fully. The answer to our question is to be found in our faith in the *uniformity of nature.* Our whole intellectual life, as well as our practical activity, is based upon the assumption that past experience may be taken as an index of future experience. Just as we expect the kind gentleman who has given us pennies to give us some more the next time we meet him, so we expect two things which have manifested certain points of resemblance to reveal other elements of likeness, if the investigation is continued. Either expectation may lead to disappointment; but we are so constructed that, even in spite of disappointment, we retain the tendency to expect.

We retain the tendency to regard resemblance in certain respects as a probable sign of further resemblance because the hypothesis "works." And the simplest explanation of its working is the assumption that our world is *a world in which resemblances go in flocks.* There is, however, no *a priori* reason why this should be so. Conceivably resemblances might not go in flocks. Conceivably the world might have been constructed in the fashion suggested by the following diagram,—only in that case it is pretty clear that *nothing*

[1] Chap. XIV.

would be conceivable. The capital letters represent objects, or complexes of qualities, and the small letters indicate the qualities belonging to, or constituting these objects or complexes.

A = m, n, r, s, w, x, etc.
B = n, o, s, t, x, y, etc.
C = o, p, t, u, y, z, etc.
D = p, m, u, r, z, w, etc.

Observe that if qualities were distributed in this way classification would be impossible, or, at least, valueless, inasmuch as the discovery of resemblance between two things would give rise to no expectation of further resemblance. While A and B are alike in the possession of n, s, and x, they differ in the remaining qualities so far as these are given; and the same sort of relation, or absence of relation, holds between B and C, C and D, etc. In a world in which objects stood to each other in this way no argument from analogy would be possible. With this, however, compare the following, which may symbolize the relations actually obtaining in our world:

A = h, i, j, k, l, m, n, o, p, etc.
B = i, j, k, l, m, n, o, p, q, etc.
C = m, n, q, r, s, t, u, v, w, etc.
D = n, q, r, s, t, u, v, w, x, etc.

In this scheme A and B are alike in the possession of the qualities i, j, k, l, m, n, o, and p, while they differ in the remaining qualities so far as these are given. In this case there is a sufficient tendency toward a grouping of resemblances to justify reasoning from analogy. In other words, in spite of a few points of difference, they are more likely than not to agree in respect to qualities not included in the enumeration. And if there should be other things which share a sufficient number of the qualities common to A and B, they together with A and B might be taken as members of a class,

and the qualities common to all would then constitute the essence or nature of this class. In the same way C and D might be put into a class, but it would not be profitable to include them in the same class with A and B.

Natural vs. Artificial Grouping.—If the individuals belonging to a group have a common nature or essence, they are called *natural* kinds or classes. Thus *man, horse,* etc. are natural kinds. Classes may, however, be delimited by an arbitrary definition. Suppose we should arbitrarily select some one attribute, such as the color of the hair or the shape of the nose, as the basis of classification. In this way we might define the class of "persons with auburn hair," or the class of "persons with a long nose." But these would be purely arbitrary groupings, and common membership in one of them would not afford any ground for inference, unless it could be shown that color of hair and length of nose are indices of other qualities. But, if this could be shown, these groupings would no longer be artificial, but natural.

Extension and Intension.—Which of the following classes contain the greatest number of members? The next greatest? The next?

Polygon, quadrilateral, rectangle, square.
Horse, animal, quadruped, vertebrate.
Evergreen tree, tree, pine tree, white pine.

About which class in each series can you make the greatest number of significant statements?

By the *extension* of a class (or a term) is meant the range or extent which it covers. By its *intension* is meant the sum of the attributes which may be predicated of it. Thus *quadruped* has a greater extension than *horse,* for *quadruped* includes all the horses and also the cows, sheep, etc. On the other hand, *horse* has a greater *intension* than quadruped, for we can affirm of a horse everything which can be affirmed

of a quadruped, and more besides. Observe that in the series of terms given in the preceding paragraph, as the extension diminishes the intension increases, and *vice versa.* The words "denotation" and "connotation" are commonly used as synonyms respectively of extension and intension.

Exercises.—Which term of each pair has the greater extension? The greater intension?

1. American, Pennsylvanian.
2. Triangle, polygon.
3. Odd number, number.
4. Periodical, Quarterly.
5. Parisian, Frenchman.
6. Blank book, book.

Genus, Species, and Differentia.—Classes such as those which we have just considered, one of which includes the other and more besides, are said to be related to each other as *genus* to *species.* Usually a genus includes several species. Thus as classified by botanists, the rose genus contains dwarf wild rose species, sweetbriar species, India rose species, and Damask rose species. The use of terms in botany is not, however, quite the same as in logic. In botany the class which stands next to the individual is called a *variety;* and above the variety we then have the *species,* the *genus,* the *family,* and the *order,* in the sequence named. In logic, however, genus and species are *relative terms.* Any class is called a genus with respect to a class which *it includes;* a species with respect to the class which *includes it.* A class which has no subclass is called an *infima species,* i.e., lowest species; while the class which is not itself a species of any other is called the *summum genus,* i.e., the highest genus. There is obviously but one *summum genus,* inasmuch as everything whatsoever may be subsumed under the genus *being or thing.*

We have already remarked that when terms are arranged in such an ascending series or "hierarchy" there is a reciprocal relation between their extension and intension. The

greater the extension the less the intension, and *vice versa.* Thus the intension of horse is greater than that of quadruped, because we may affirm of a horse all the qualities which it shares with all other quadrupeds, plus those which are peculiar to the horse. This additional meaning possessed by the species, as compared with its genus, is known technically as the *differentia.* In other words, the differentia is the difference between the intension of the species and that of its genus.

Division.—The construction of a classification by the enumeration of all the species which fall under each of the several genera is known as *division.* In order that division may be logically correct certain rules must be observed:

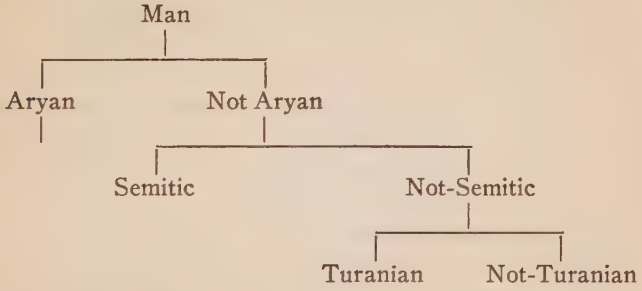
1. The several species which fall under a given genus should include all the individuals which are included by the genus. In other words, the division should be *exhaustive.* For example, the division of Europeans into Germans, Russians, Italians, and Frenchmen, would obviously be unsatisfactory.

2. Not only should all the species be enumerated, but they should be *mutually exclusive.* There should be no overlapping. For example, the division of the population of Pennsylvania into Caucasians, Negroes, Germans, Scotch, Irish, Farmers, Merchants, Presbyterians, Methodists, and Roman Catholics, is unsatisfactory. For the same individual might be a member of two or more of these classes. This error may be avoided by seeing to it that there is but one *"fundamentum divisionis,"* that is to say, that one ground or principle of division is adhered to throughout. Thus in our example, if Pennsylvanians are divided on the basis of *race,* they might be classified as Caucasian, Negro, Mongolian, American Indian, etc. If the basis of the classification is *occupation,* the species would be farmers, merchants, miners, lawyers, teachers, etc. While if the basis is *religious*

affiliation, they would be Presbyterians, Methodists, Roman Catholics, Lutherans, Reformed, Baptists, Episcopalians, etc.

If then any genus, e.g., M, is correctly divided into the species A, B, C, D, and if x is any member of the class M, we know that x is *either* a member of A, of B, of C, *or* of D. In other words, we obtain a disjunctive proposition; for the alternatives are exhaustively enumerated, and they are completely disjoined one from the other.

Dichotomy.—A classification in which each genus is divided exhaustively into *two* mutually exclusive species is known as a "dichotomy." The following example is given by Jevons:



Definition.—In a previous chapter we discussed the problem of *definition.* It is possible now to give that discussion a somewhat more precise statement. There are two senses in which we may speak of the defining of a term. From the first point of view, definition is an attempt to mark off the limits of the class which is denoted by the term in question; from the second point of view, the purpose of definition is to specify the essential attributes of the class. The former is definition from the standpoint of *extension;* the latter from the point of view of *intension.* The former kind of definition is then identical with what we have been discussing under the name of division; for a term has been defined in

this sense when its class has been exactly placed as a species of a certain genus. The intensive sort of definition, which, by the way, is what is usually meant when we speak without qualification of the definition of a term, is also better understood when it is related to the idea of classification. From this point of view,

DEFINITION = GENUS + DIFFERENTIA

This means that a term is defined when the class which it denotes is shown to be distinguished from other species of the same genus by the possession of certain attributes. In this *definition of definition,* however, a *class* is understood to be not merely a group of objects possessing certain common characteristics, but rather the *nature* possessed in common by all the members of the class. In other words, both species and genus, from this point of view, are "universals" or logical "essences." The *specific* nature or essence, however, is richer in content than the generic, and this additional content is what is called the *differentia.*

Study the following examples of analogy. In each case, what is the conclusion which is to be established, and upon what resemblances does it depend? Can you find important points of difference which destroy the force of the analogy?

Exercises.—1. The speaker was advocating preparedness. "I will not say," he shouted, "that you should lock the stable before the horse is stolen, but rather that you should call the fire company before the fire breaks out."

2. As an argument for an adequate support of the Christian ministry it is urged that it is "even more important to provide food for the soul than it is to provide food for the body."

3. A United States Congressman is sent to Washington to represent his constituents. He is, so to speak, their "hired man." Therefore, in casting his vote upon measures in which his constituents are interested, he should be controlled by their judgment concerning public policy rather than by his own.

4. A Congressman stands to his constituents in much the same relation as a lawyer to his client, or a physician to his patient. He is chosen because he is supposed to know more than they about the questions which come before the Congress for decision.

5. Age is wiser than youth; therefore we should accept the decisions of our ancestors.

6. Ruminant animals have cloven feet, and they usually have horns; the extinct animal which left this foot-print had a cloven foot; therefore it was a ruminant animal and had horns. (Jevons.)

7. Italy is a Catholic country and abounds in beggars; France is also a Catholic country, and therefore abounds in beggars. (Ibid.)

8. Before it was known that light traveled in waves, it was known that sound did so. Light and sound were both capable of being reflected, and the direction of their reflection obeyed the same law, that the angle of reflection is equal to the angle of incidence. From these facts it was inferred that light, like sound, traveled in waves. (Creighton.)

9. The community cannot possibly govern itself while its parts are separated. . . . Now our 'residential' sections are separated from the rest of our cities, and are often the most useless. The people in the 'residential' sections are those who dominate our big business interests. And yet they are out of the current of civilization. There is the atrophy of the body politic. (Woodrow Wilson, quoted by Creighton.)

10. No body can be healthful without exercise. And certainly to a kingdom or an estate, a just and honorable war is the true exercise. A civil war, indeed, is like the heat of a fever; but a foreign war is like the heat of exercise, and serveth to keep the body in health. (Francis Bacon.)

11. Children, until they reach the age of discretion, must be cared for and controlled by their parents and teachers. In the same way backward and undeveloped nations should be subject to the control of those which are more advanced.

12. Ye are the salt of the earth. But if the salt have lost its savor, wherewith shall it be salted.

13. The kingdom of heaven is like unto a treasure hidden in the field; which a man found, and hid; and in his joy he goeth and selleth all that he hath, and buyeth that field.

14. What woman, having ten pieces of silver, if she lose one piece, doth not light a lamp, and sweep the house, and seek diligently until she find it? And when she hath found it, she calleth together her friends and neighbors, saying, Rejoice with me, for I have found the piece which I had lost. Even so, I say unto you, there is joy in the presence of the angels of God over one sinner that repenteth.

15. For the body is not one member, but many. If the foot shall say, Because I am not the hand, I am not of the body; it is not therefore not of the body. And if the ear shall say, Because I am not the eye, I am not of the body; it is not therefore not of the body. If the whole body were an eye, where were the hearing? If the whole body were hearing, where were the smelling? But now hath God set the members each one of them in the body, even as it pleased him. And if they were all one member, where were the body? But now they are many members, but one body. And the eye can not say to the hand, I have no need of thee: or again the head to the feet, I have no need of you. . . . But God tempered the body together . . . that there should be no schism in the body; but that the members should have the same care one for another. And whether one member suffereth, all the members suffer with it; or one member is honored, all the members rejoice with it. Now ye are the body of Christ, and severally members thereof.

16. We should think it a sin and a shame if a great steamer, dashing across the ocean, were not brought to a stop at a signal of distress from a mere smack. . . . And yet a miner is entombed alive, a painter falls from a scaffold, a brakeman is crushed in coupling cars, a merchant fails, falls ill, and dies, and organized society leaves widow and child to bitter want or degrading alms. (Henry George, quoted by Creighton.)

17. If the Prohibitionists want to prohibit everything that has evil in it, let them be consistent and stop not at alcohol, but go a little further and include the human tongue, of which the Bible says, 'The tongue can no man tame. It is an unruly evil full of deadly poison; therewith we bless God, even the Father, and therewith curse we men made after the similitude of God.' (James iii: 8, 9.) Here is evil and good combined in the same thing, just owing to whether it is properly or improperly used,

and the same is equally true of alcohol. (*The Arena,* Vol. 8, p. 205, quoted by Bode.)

18. The attempt to prohibit the sale of wine and beer, because some foolish people have abused the use of alcohol, is an interference with the liberty of those who are able to control themselves. For the same reason no laws should be passed to limit the use of opium. If the use of alcohol and opium is to be regulated or prohibited, the same should be done with the use of pie and cake.

19. Society has no more right to expose its members to the dangers arising from the consumption of alcohol than a parent has to allow his children to play with a loaded revolver.

20. Just as the thirteen colonies came together to form the United States of America, so the nations of the world will eventually unite to form a world state.

21. A parricide, said David Hume, is in the same relation to his father as is the parent tree to a young oak, which, springing from an acorn dropped by the parent, grows up and overturns it; we may search as we like, but we shall find no vice in this event; therefore there can be none in the other, where the relations involved are just the same; so that it is not until we look beyond to the feelings with which other persons regard it, that we can find the ground for calling it vicious. Doubtless there is an analogy here; but the relations are not altogether the same; for the relation of a parent to a child is spiritual as well as physical, and in the parricide there is an attitude of the will and the affections which cannot be ascribed to the oak. (Joseph, *Introduction to Logic,* p. 534.)

22. It has been objected to hedonistic systems that pleasure is a mere abstraction, that no one could experience pleasure as such, but only this or that species of pleasure, and that pleasure is therefore an impossible criterion. . . . It is true that we experience only particular pleasurable states which are partially heterogeneous with one another. But this is no reason why we should be unable to classify them by the amount of a particular abstract element which is in all of them. No ship contains abstract wealth as a cargo. Some have tea, some have butter, some have machinery. But we are quite justified in arranging those ships, should we find it convenient, in an order determined by the extent to which their concrete cargoes possess the

abstract attribute of being exchangeable for a number of sovereigns. (McTaggart, *Studies in Hegelian Cosmology,* quoted by Joseph, p. 534.)

23. Two jolly scholars, whom I like to see at my house, returned from the country one day, and described with youthful enthusiasm the pleasure they had had. One of them smiled as if he had a good joke up his sleeve, and told in these words the day's adventure:

"In the forest we came to a pond, in the middle of which was a small island. 'I bet you can't jump that ditch,' said one of my sly companions, and the others urged me on with an eagerness I ought to have suspected. Sure of my ability, I accepted the challenge. I jumped boldly and landed in the middle of the island and turned triumphantly to my companions. To my surprise I saw them doubled up with laughter, and when I asked the cause they shouted in chorus, 'Now come back.' I hadn't noticed that the island was too small to allow me to get a start, and so I was obliged to wade ashore." Is not that a picture of life, of the thoughtless conduct we often see? We throw ourselves into adventures, carried away by pleasure, by pride—in a word, by our passions—and we do not realize that if we get out at all it will only be with a splashing. (DuBois, *Self-Control and How to Secure It,* p. 9.)

24. If a merchant commenced business without any knowledge of arithmetic and book-keeping, we should exclaim at his folly, and look for disastrous consequences. Or if, before studying anatomy, a man set up as a surgical operator, we should wonder at his audacity and pity his patients. But that parents should begin the difficult task of rearing children without ever having given a thought to the principles—physical, moral, or intellectual—which ought to guide them, excites neither surprise at the actors nor pity for their victims. (Spencer, *Education,* p. 45.)

25. It is a general law of life that the more complex the organism to be produced, the longer the period during which it is dependent upon a parent organism for food and protection. The contrast between the minute, rapidly-formed, and self-moving spore of a confera, and the slowly developed seed of a tree, with its multiplied envelopes and large stock of nutriment laid by to nourish the germ during its first stages of growth, illustrates this law in its application to the vegetable world.

Among animal organisms we may trace it in a series of contrasts from the monad whose spontaneously-divided halves are as self-sufficing the moment after their separation as was the original whole; up to man, whose offspring not only passes through a protracted gestation, and subsequently long depends on the breast for sustenance; but after that must have its food artificially administered; must, after it has learned to feed itself, continue to have bread, clothing, and shelter provided; and does not acquire the power of complete self-support until a time varying from fifteen to twenty years after its birth. Now this law applies to the mind as to the body. For mental pabulum also, every higher creature, and especially man, is at first dependent upon adult aid. Lacking the ability to move about, the babe is as powerless to get materials on which to exercise its perceptions as it is to get supplies for its stomach. Unable to prepare its own food, it is in the same manner unable to reduce many kinds of knowledge to a fit form for assimilation. The language through which all higher truths are to be gained it wholly derives from those surrounding it. And we see in such an example as the Wild Boy of Aveyron, the arrest of development that results when no help is received from parents and nurses. Thus, in providing from day to day the right kind of facts, prepared in the right manner, and giving them in due abundance at appropriate intervals, there is as much scope for active ministration to a child's mind as to its body. In either case it is the chief function of parents to see that the *conditions* requisite to growth are maintained. And, as in supplying aliment, and clothing and shelter, they may fulfil this function without at all interfering with the spontaneous development of the limbs and viscera either in their order or mode; so they may supply sounds for imitation, objects for examination, books for reading, problems for solution, and if they use neither direct nor indirect coercion, may do this without in any way disturbing the normal process of mental evolution; or rather, may greatly facilitate that process. (*Ibid.*, p. 114.)

26. Of errors in education one of the worst is that of inconsistency. As in a community crimes multiply when there is no certain administration of justice; so in a family an immense increase of transgressions results from a hesitating or irregular infliction of penalties. (*Ibid.*, p. 227.)

27. We may observe a very great similitude between the earth which we inhabit and the other planets. They all revolve around the sun as the earth does, though at different distances and in different periods. They borrow all their light from the sun as the earth does. Several of them are known to revolve round their axis like the earth, and by that means have a like succession of day and night. Some of them have moons that serve to give them light in the absence of the sun, as our moon does to us. They are all, in their motions, subject to the same law of gravitation as the earth is. From all this similitude, it is not unreasonable to think that these planets may, like our earth, be the habitation of various orders of living creatures. (Reid, *Intellectual Powers*, quoted by Hibben.)

28. Colonies ought not to rebel against the mother country, since they are its children and children ought not to rebel against their parents.

29. Standing under the shadow of the doorway, at the extreme end of the room, he saw a little figure watching him. . . . It was a monster, the most grotesque monster he had ever beheld. Not properly shaped, as all other people were, but hunchbacked, and crooked-limbed, with large lolling head and mane of black hair. The little dwarf frowned, and the monster frowned also. He laughed, and it laughed with him, and held its hands to its sides, just as he himself was doing. He made it a mocking bow, and it returned him a low reverence. He went towards it, and it came to meet him, copying each step that he made, and stopping when he stopped himself. He shouted with amusement, and ran forward, and reached out his hand, and the hand of the monster touched his, and it was as cold as ice. He grew afraid, and moved his hand across, and the monster's hand followed it quickly. He tried to press on, but something smooth and hard stopped him. The face of the monster was now close to his own, and seemed full of terror. He brushed his hair off his eyes. It imitated him. He struck at it, and it returned blow for blow. He loathed it, and it made hideous faces at him. He drew back, and it retreated.

What is it? He thought for a moment, and looked round at the rest of the room. It was strange, but everything seemed to have its double in this invisible wall of clear water. Yes, picture for picture was repeated, and couch for couch. The sleeping

Faun that lay in the alcove by the doorway had its twin brother that slumbered, and the silver Venus that stood in the sunlight held out her arms to a Venus as lovely as herself.

Was it Echo? He had called to her once in the valley, and she had answered him word for word. Could she mock the eye as she mocked the voice? Could she make a mimic world just like the real world? Could the shadows of things have color and life and movement? Could it be that——?

He started, and taking from his breast the beautiful white rose, he turned round, and kissed it. The monster had a rose of its own, petal for petal the same! It kissed it with like kisses, and pressed it to its heart with horrible gestures.

When the truth dawned upon him, he gave a wild cry of despair, and fell sobbing to the ground. So it was he who was misshapen and hunchbacked, foul to look at and grotesque. He himself was the monster! (Wilde, *The Birthday of the Infanta.*)

30. Which of the following conform to the principles of classification?

(a) Some women are pretty, some are attractive, and some are witty.

(b) The people of Europe may be divided into peasants, workingmen, Protestants, and Italians.

(c) Books may be classified as bound and unbound.

(d) The population of the United States may be divided into the foreign-born and the native.

(e) Men are either good or bad.

31. Which term of each pair has the greater intension?

(a) Cow, quadruped; (b) triangle, polygon; (c) Animal, bird.

32. Define the following terms by giving the genus and differentia.

(a) Tree; (b) Rectangle; (c) Crime.

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CHAPTER XII

SCIENTIFIC METHOD IN GENERAL

1. In a right triangle the square of the hypotenuse is equal to the sum of the squares of the other sides.

2. The circumference of a circle is equal to the diameter multiplied by 3.14159.

3. $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{1.2} a^{n-2}b^2 \ldots\ldots$

4. $a+b = b+a$

5. $2+2 = 4$

Can you prove the correctness of the above statements? Can we prove them inductively? Let us see. Consider the first. We might draw a great many right triangles and measure their several sides. We might then seek for some formula which would give the indicated relation. The theorem may perhaps have been discovered in this manner. The following are the results of actual measurement:

First Leg Inches	Second Leg Inches	Hypotenuse Inches
20	30	36.00
24	25	34.75
25	35	43.25
30	40	49.87
32	45	55.12
36	48	59.50

The students who made these measurements did not stop to calculate what the length of the hypotenuse in each case *ought* to be, but read off the results as accurately as they

could from an ordinary yard-stick. Now if we may suppose a person who is ignorant of the theorem concerning the relation of the hypotenuse to the other sides of a triangle to have made these rough measurements and many others of the same sort, he would at first be at a loss to find any satisfactory statement of the relation. If, after a while, he were so fortunate as to think of the formula $a^2 + b^2 = c^2$, this would come to him as an inspiration, a happy thought, a *hypothesis* to be tested by subsequent measurements. If, however, he found that new measurements were approximately in agreement with the results required by the formula, and that the approximation became closer and closer the more accurately the measurements were made, he would be justified in regarding the hypothesis as established.

In much the same way a ratio closely approximating 3.1416 might be obtained by measuring a sufficiently great number of circles, dividing the circumference in each case by the diameter, and taking the average of the quotients thus obtained.

Empirical versus Demonstrative Proof.—It is in the manner just described that many of the values employed in the physical sciences have been discovered. For example, to learn the specific heat of mercury, or the atomic weight of aluminum, or the food value of wheat, one would make a great many very careful measurements, and then average the results. The *mathematician,* however, would not be satisfied with such empirical results. He does not depend upon a *multitude of measurements* to obtain the relation of the hypotenuse to the legs of the right triangle, or of the circumference to the diameter of the circle. He believes himself to be in possession of a better way. This better way of the mathematician is what we know, in a special sense of the word, as *demonstration.* Mathematical demonstration is essentially deductive; but, as we shall see, it includes non-

deductive elements also. It is hard to give an idea of the meaning of rigorous mathematical thinking to one who has not experienced it. The joy of an absolutely cogent nexus of thought, is found, perhaps best of all, in geometry. For the sake of those who have not rightly entered into the spirit of geometrical demonstration—who, perhaps, have *memorized* the proof of the theorems without really perceiving the logical nexus—we shall discuss in some detail an interesting example. This should not, however, be considered a digression. On the contrary our account of scientific method would otherwise be incomplete. For *mathematics is the essence of science, and as the special sciences become more genuinely scientific, they become more and more mathematical.*

Demonstration of the Pythagorean Theorem.—The governing principle of demonstrative reasoning is the attempt to build up a body of interrelated propositions, every one of which, except those which are called "axioms," "postulates," and "definitions," can be shown to follow by logical necessity from other propositions which have already been logically established. It is manifest that *not all* propositions can be proved in this way; for the demonstration must start somewhere. Certain premises must be taken for granted. Now these propositions which are thus *begged* or regarded as *self-evident* are the postulates and axioms. The mathematician, however, seeks to organize his science so as to reduce the number of axioms and postulates as much as possible. He strictly obeys the "law of parsimony," so far as the admission of unproved propositions is concerned. In our effort to appreciate the spirit of mathematical reasoning, let us study the demonstration of the Pythagorean theorem, —so-called because it is said to have been known, perhaps to have been discovered, by Pythagoras a famous Greek philosopher who died about five hundred years before Christ.

At least two demonstrations of this theorem are given by mathematicians. One of these is the following:

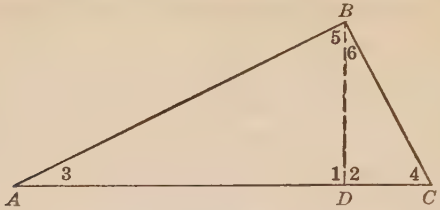


FIG. 5.—THE PYTHAGOREAN THEOREM.

Let ABC be a right triangle, with the right angle at B. From B drop a perpendicular BD upon the side AC. We are to prove that $AC^2 = AB^2 + BC^2$. To prove this equality, we quote a previous theorem to the effect that if in a right triangle a perpendicular is drawn from the vertex of the right angle to the hypotenuse, each leg of the triangle is the mean proportional between the entire hypotenuse and its adjacent segment. This means that in our triangle,

$$AB^2 = AC \times AD$$

and, $$BC^2 = AC \times CD$$

And now, if we add both sides of these equations, we find that the sum of the squares of the legs is equal to the square of the hypotenuse, which was the proposition to be proved.

But now the question arises, how can we justify our use of the theorem that each leg is the mean proportional between the hypotenuse and the adjacent segment? To prove this we quote a previous theorem to the effect that if two triangles are mutually equiangular, their corresponding sides are proportional. Applying this theorem to our figure, it is evident that

$\angle 1 = \angle B$ (since both are right angles)
$\angle 3 = \angle 3$ (by identity)
$\angle 5 = \angle 4$ (since the interior angles of a triangle are

equal to two right angles, and if equals are subtracted from equals the remainders are equal.) Triangles ABD and ABC are therefore mutually equiangular, and by the theorem quoted, $AB : AC = AD : AB$. Hence (by the principle of proportion) $AB^2 = AC \times AD$.

In the same way we can prove that triangles BDC and ABC are mutually equiangular, and that $BC^2 = AC \times CD$. But now the plot thickens; for we have quoted several propositions which we have not proved. To run a theorem back, in this fashion, to axioms, postulates, and definitions, is evidently a very complex undertaking. I shall therefore exhibit the work in a summary form, which, I hope, will nevertheless be readily understood.

1. If a perpendicular is drawn from the right angle to the hypotenuse as base, each leg is the mean proportional between the entire hypotenuse and the adjacent segment.

1.1.—If two triangles are mutually equiangular, their corresponding sides are proportional.

1.11.—The principle of superposition. (Postulate.)

1.12.—When two straight lines in a plane are cut by a transversal, if the exterior-interior angles are equal, the two straight lines are parallel.

1.121.—Through a given point a line may be drawn to a given straight line. (Postulate.)

1.122.—If two parallel lines are cut by a transversal, the exterior-interior angles are equal.

1.1221.—Vertical angles are equal.

1.12211.—If two adjacent angles have their exterior sides in the same straight line, they are supplementary. (See 1.514.)

1.12212.—All straight angles are equal. (See 1.41.)

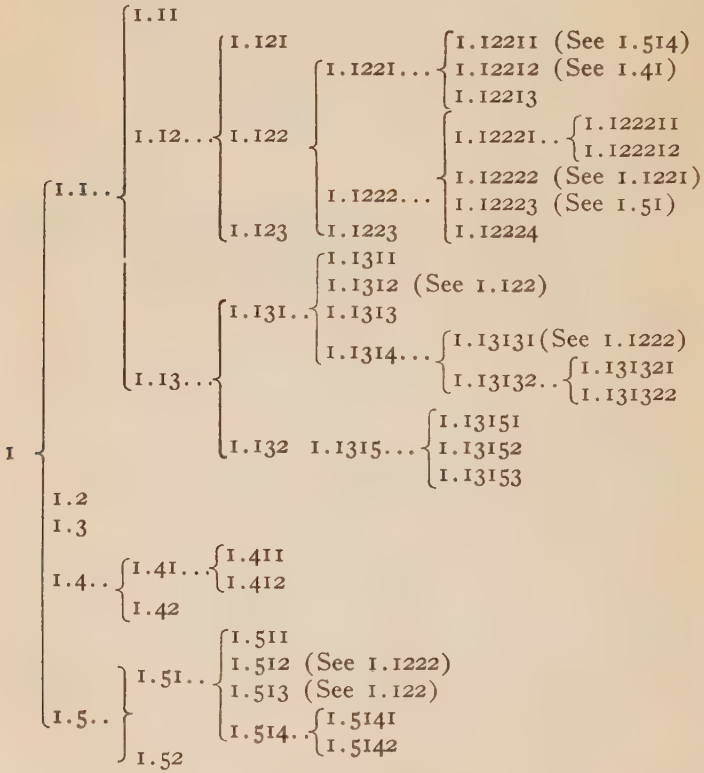
1.12213.—If equals be subtracted from equals, the remainders are equal. (Axiom.)

1.1222.—Alternate-interior angles are equal.

1.12221.—If a straight line is perpendicular to one of two parallels, it is perpendicular to the other.

1.122211.—Two straight lines in the same plane perpendicular to the same straight line are parallel. (Axiom.)

A diagram may help to show the relation of the propositions. The theorem depends upon propositions 1, 2, 3, and 4. Of these, 2, 3, and 4 require no proof; while 1 may be established in the manner indicated in the diagram. In each case the propositions to the *right* of the brace support the one to the *left*. The logical relations are also suggested by the numbers given to the propositions. For example, 1.12 is supported by 1.121, 1.122, and 1.123.



1.122212.—The possibility of constructing the figure. (Postulate.)

1.12222.—Vertical angles are equal. (See 1.1221.)

1.12223.—The sum of the interior angles of a triangle is equal to two right angles. (See 1.51.)

1.12224.—If equals be subtracted from equals, the remainders are equal. (Axiom.)

1.1223.—Equals of the same are equal to each other. (Axiom.)

1.123.—Things equal to the same thing are equal to each other. (Axiom.)

1.13.—If a line is drawn through two sides of a triangle parallel to the third side, it divides those sides proportionally.

1.131.—If three or more parallels intercept equal parts on one transversal, they intercept equal parts on every transversal.

1.1311.—Through given points it is possible to draw lines parallel to a given line. (Postulate.)

1.1312.—Exterior-interior angles are equal. (See 1.122.)

1.1313.—The opposite sides of a parallelogram are equal.

1.13131.—Alternate-interior angles are equal. (See 1.1222.)

1.13132.—Two triangles are equal if two sides and the included angle of the one are equal respectively to two sides and the included angle of the other.

1.131321.—The possibility of constructing the figure. (Postulate.)

1.131322.—Two straight lines can intersect in only one point. (Axiom.)

1.1314.—Equals of the same are equal to each other. (Axiom.)

1.1315.—If two triangles have a side and the two adjacent angles of the one equal to a side and the two adjacent angles of the other, the two triangles are equal.

1.13151.—One figure may be applied to another. (Postulate.)

1.13152.—Two straight lines can intersect in only one point. (Axiom.)

1.13153.—Coincidence is a test of equality. (Axiom.)

1.132.—Equals of the same are equal to each other. (Axiom.)

1.2.—The product of the means is equal to the product of the extremes. (The principle of proportion.)

1.3.—Any given quantity is equal to itself. (Axiom.)

1.4.—All right angles are equal.

1.41.—All straight angles are equal.

1.411.—One may be applied to the other. (Postulate.)

1.412.—Coincidence is an evidence of equality. (Axiom.)

1.42.—The halves of equals are equal. (Axiom.)

1.5.—If two angles of one triangle are equal respectively to two angles of another triangle, the third angle of the first is equal to the third angle of the second.

1.51.—The sum of the interior angles of a triangle is equal to two right angles.

1.511.—Through a given point a line may be drawn parallel to a given line. (Postulate.)

1.512.—Alternate-interior angles are equal. (See 1.1222.)

1.513.—Exterior-interior angles are equal. (See 1.122.)

1.514.—If two adjacent angles have their exterior sides in the same straight lines, these angles are supplementary.

1.5141.—The whole is equal to the sum of its parts. (Axiom.)

1.5142.—The possibility of constructing the figure. (Postulate.)

2.—It is possible to draw a perpendicular from a given point to a given line. (Postulate.)

3.—If equals are added to equals the sums are equal. (Axiom.)

4.—$ab + ac = a(b + c)$. (Law of Composition.)

Exercises.—1. The foregoing demonstration may not be completely satisfactory. Criticise it. How many distinct axioms or postulates are required for the proof of the Pythagorean theorem?

2. In the same way,—leaving no proposition unproved except those which are presented as axioms, postulates, or definitions,—give a complete demonstration of the following:

(a) Every angle inscribed in a semi-circle is a right angle.

(b) The side of a regular hexagon is equal to the radius of the circumscribed circle.

Non-Deductive Elements in Geometrical Demonstration.—For the most part, geometrical demonstration is deductive. The argument could be thrown into a series of syllogisms. Nevertheless there are certain elements which cannot be established deductively. We have already spoken of the axioms." "An axiom," some one has said, "is a truth which is self-evident to all persons of sound mind."

Yet what has appeared to be axiomatic to some minds, may not seem true at all to minds which are presumably just as sound. Certain of Euclid's axioms are rejected by some modern geometers, who nevertheless succeed in building up a geometry just as rigorous as that of Euclid. Perhaps the propositions which have traditionally been considered as axioms should be included among the postulates; i.e., among the propositions which we cannot prove, but which we find it convenient to assume without proof. In addition to the axioms we make certain postulates, as that one figure can be applied to another, or that between any two points a straight line can be drawn. However, as we have previously remarked, the geometer tries to reduce the number of axioms and postulates as much as possible.[1]

Moreover, in geometrical demonstration some things are said to be true "by definition." And definitions, too, must be accepted as requiring no proof. The geometer seeks to define in such a way as to have the smallest possible number of *indefinables*. Every concept is defined in terms of something simpler, until at length we come to concepts which are incapable of further analysis. Philosophical students of mathematics have sought to reduce these indefinable, or not-further-analyzable concepts, to a very few; for example to the concepts "point," "between," and one or two others.

The Facts of Arithmetic.—How do you know that the multiplication table is true? Perhaps multiplication can be shown to be only a more complex form of addition. But how can we know that our addition is true. How do we know, for example, that $2 + 2 = 4$? We may be tempted to answer, "From experience." We count beans or toothpicks, and always get this result. Thus J. S. Mill said that

[1] It must be admitted, I think, that all axioms are postulates. And yet there seems to be a difference. Some of the postulates are unavoidable. We can not treat them as theorems. Other postulates may become theorems if some theorems are made postulates in their stead.

in another world two and two might be five. Yet this is not the true view of the matter. Indeed, we hold that two and two make four *in spite of experience* as well as because of it. For example, two drops of water and two other drops of water on the window-pane may flow together to make *one*. On the other hand, if two pieces of glass are added (with sufficient violence) to two other pieces of glass, they may make ten or a hundred. Evidently the laws of arithmetic hold only on the assumption that the *units* with which we deal are *permanent*.

Proof that $2 + 2 = 4$.—It is, then, impossible to prove inductively, or by any appeal to experience, that $2 + 2 = 4$. Can it be proved deductively? The philosopher Leibniz gave a very interesting demonstration.[1] It will repay study. It is substantially as follows:

Let us *assume* that we know the meaning of "1" or unity, and also of the operation of addition.

Let us furthermore agree upon the following definitions:

$$\text{“}2\text{”} = 1 + 1$$
$$\text{“}3\text{”} = 2 + 1$$
$$\text{“}4\text{”} = 3 + 1$$

We are now to prove that $2 + 2 = 4$.

Proof:

By definition, $4 = 3 + 1$.
Consequently, $4 = (2 + 1) + 1$.
$= 2 + (1 + 1)$
$= 2 + 2$

We notice that this demonstration, like those of geometry, depends upon certain concepts which are simply postulated or taken for granted. It should be remarked, too, that there is a postulate which has not been avowed; viz., that the *order* of addition is immaterial.

[1] See Poincaré, *Science and Hypothesis*, p. 3.

Mathematical reasoning, then, goes beyond the forms of deduction and induction, in that it deals with the relations of *number*. It is *quantitative*. It includes certain *operations*, certain methods of calculation, such as addition, subtraction, multiplication, division, involution and evolution, differentiation and integration, which are not found in ordinary logic.

The Elements of Scientific Method.—Sometimes we fail to see the woods because our attention is held by the trees. The bird's-eye view, deficient though it be in point of detail, has a value of its own. Can we, then, sum up our doctrine of scientific method under a few main heads? As a tentative classification of the many types of method employed by the scientist, I would suggest the following:

ELEMENTS OF SCIENTIFIC METHODS

1. ANALOGY (including Classification and Syllogism.)
2. HYPOTHESIS (including Inductive Methods, Statistics, etc.)
3. MATHEMATICS.

I do not mean to suggest that analogy, hypothesis, and mathematics are mutually exclusive. On the contrary it is easy to show that they overlap. Nevertheless hypothesis contains an element not found in analogy, classification, or syllogism; and mathematics includes processes which cannot be subsumed under either analogy or hypothesis. Indeed, if the term *mathematics* is broadly interpreted, it may be made to include all that is ordinarily considered to belong to logic. For mathematics may be considered as the *pure science of order*, of which logic would then be a subdivision.

Exercises.—What elements of scientific method,—syllogism, classification, analogy, statistical reasoning, induction, hy-

pothesis, mathematical reasoning, etc.—can you point out in the following:[1]

1. Aristotle attempted to determine whether air had weight by weighing a bladder collapsed and then inflated. Of course the change of buoyancy in the two cases offset the difference due to the weight of the air removed.

Galileo determined that water would not rise above 32 feet in the pumps of the Duke of Tuscany. . . . The total pressure of the atmosphere was first discovered by Torricelli in 1643. He took a glass tube, a little less than a meter long and closed at one end, and filled it with mercury. Closing the upper end with the thumb he inverted the tube and placed the lower end under mercury contained in another vessel. On removing the thumb the mercury fell in the tube, and came to rest at a height of 76 cms. above the mercury surface in the outer vessel, leaving a vacuum in the tube above it, which has since been known as the Torricellian vacuum. It was rightly concluded by Torricelli that the mercury is sustained in the tube by the pressure of the atmosphere on the mercury surface external to the tube.

Pascal performed two experiments to demonstrate that the column of mercury is supported by atmospheric pressure. In the first the mercury was replaced by lighter liquids, with the result that the height of the sustained column was always inversely proportional to the density of the liquid.

In the second experiment Pascal had the mercury carried to the top of the Puy-de-Dome, about 1,000 metres high. The pressure of the atmosphere being less on top of the mountain, it was anticipated that the mercury column would fall. A fall of nearly eight centimetres was observed. (Carhart, *University Physics.*)

2. The emission theory requires that light travel faster in the denser medium, like water or glass, than in the rarer; the undulatory theory leads to the opposite conclusion. Foucault's experiment showed conclusively that the emission theory is untenable, for light travels slower in water than in air. (*Ibid.*)

3. Newton's conclusion that refraction does not produce the colors, but serves only to separate those already mingled in white light, was confirmed by recombining the separated colors

[1] No attempt has been made to select these examples of scientific reasoning so as to illustrate this, that, or the other mode of argument. I hope that the reader will realize that approved scientific method is much *richer* than any account of it which logic can give.

into a beam of white light. One of the methods employed was the reflection to one point by seven mirrors of the different rays of the spectrum. The superposition of these produced white light. But a more conclusive experiment was the actual synthesis of white light from the spectral colors by means of a second prism identical with the first and placed with its refracting edge turned in the opposite direction. (*Ibid.*, p. 286.)

4. While some of the early philosophers surmised that the lightning flash was an electrical discharge, yet this view obtained but little currency till Franklin's suggestion in 1749 to apply his discovery of the discharging power of points to the investigation of the problem had actually been carried into effect. In 1752 d'Alibard, acting on Franklin's suggestion, erected an iron rod 40 feet high, but not connected with the earth, and drew sparks from passing clouds. About the same time (1752) Franklin sent up his famous kite by means of a linen thread, during a thunder storm, and held it by means of a silk ribbon between his hand and a key attached to the thread. When the thread had been wetted by the rain, sparks were drawn from the key and a Leyden jar was charged.

5. After Franklin had investigated the nature of electricity for some time, he began to consider how many of the effects of thunder and lightning were the same as those produced by electricity. Lightning travels in a zigzag line, and so does electricity; electricity sets things on fire, so does lightning; electricity melts metals, so does lightning. Animals can be killed by both, and both cause blindness. Pointed bodies attract the electric spark, and in the same way lightning strikes spires, and trees, and mountain tops. Is it not likely then that lightning is nothing more than electricity passing from one cloud to another, just as an electric spark passes from one substance to another? (Buckley, *A Short History of Natural Science,* quoted by Creighton.)

6. The source of the electrification of the atmosphere and of clouds remains as yet unsettled. But given ever so slight an electrification of aqueous vapor, it is not difficult to account for the high potential exhibited by thunder clouds. Consider minute spherules of water condensing to large drops in the formation of clouds and rain. Since the volumes of spheres vary as the cubes of their radii, eight small drops condense into one of

double radius. Therefore each of the larger drops contains eight times the charge of the smaller ones. But since the capacity of a sphere is numerically equal to its radius, the larger sphere has only double the capacity of the smaller ones. Therefore its potential, which is the quotient of the charge by the capacity, is quadrupled. The potential then increases as the square of the radius of the drops. If the potential increases according to such a law, the inductive influence and tendency to discharge from drop to drop through a cloudy atmosphere rise in the same proportion. (Carhart, *University Physics,* II., 224.)

7. *The Earth a Magnet.*—Since a suspended magnetic needle tends to set itself in a definite direction, it follows that the space about the earth is a magnetic field. . . . Take a piece of gas-pipe a metre long and carefully freed from magnetism. . . . Hold it vertically, or, better still, in the meridian and inclined about 75 degrees below the horizontal toward the north, and strike it a sharp blow on the upper end with a hammer. It has now acquired permanent magnetism with the N pole at the lower end. This fact can be demonstrated by holding the pipe horizontally east and west. By reversing it and striking it on the other end the polarity may be reversed, and by graduating the strength of the blow the pipe may be nearly or quite demagnetized.

The earth acts inductively on the pipe, as any other magnet does on a piece of iron, putting it under magnetic stress. The vibration due to the blow gives a certain freedom of motion to the molecules, and they arrange themselves to some slight extent under the influence of the earth's magnetic stress. With the molecules so arranged the pipe becomes a magnet. Bars of iron or steel in a vertical or horizontal north-and-south position acquire magnetism by induction from the earth. This is especially true if they are subjected to frequent jarring. Drills, railway iron, beams and posts are illustrations. (*Ibid.,* II., 324.)

8. Rumford then designed a gun-metal cylinder for the express purpose of generating heat by friction. A blunt rectangular piece of hardened steel, which he called a borer, was forced edgeways against the solid bottom of the cylinder, while the latter was turned round its axis by the force of horses. To measure the heat developed, a small round hole was bored in the cylinder, into which was introduced a small mercurial ther-

mometer. The weight of the cylinder was 113.13 lbs. avoirdupois. The borer was 0.63 of an inch thick, 4 inches long, and nearly as wide as the cavity of the bore of the cylinder, namely 3.5 inches. . . . Rumford next surrounded his cylinder by an oblong deal box, so that the cylinder could turn water-tight in the center of the box, while the borer was pressed against the bottom of the cylinder. The box was filled with water until the entire cylinder was covered, and then the apparatus was set in motion. The temperature of the water on commencing was 60 degrees Fahrenheit.

"The result of this beautiful experiment," writes Rumford, "was very striking, and the pleasure it afforded me amply repaid me for all the trouble I had had in contriving and arranging the complicated machinery used in making it. The cylinder had been in motion but a short time, when I perceived, by putting my hand into the water, and touching the outside of the cylinder, that heat was generated.

"At the end of one hour the fluid, which weighed 18.77 lbs., or 2.5 gallons, had its temperature raised 47 degrees, being now 107 degrees.

"In thirty minutes more, or one hour and thirty minutes after the machinery had been set in motion, the heat of the water was 142 degrees.

"At the end of two hours from the beginning, the temperature was 178 degrees.

"At two hours and twenty minutes it was 200 degrees; and at two hours and thirty minutes it ACTUALLY BOILED!"

. . . "By meditating on the results of all these experiments, we are naturally," he says, "brought to the great question which has so often been the subject of speculation among philosophers, namely, What is heat—is there any such thing as an *igneous fluid?* Is there anything that, with propriety, can be called caloric?

"We have seen that a very considerable quantity of heat may be excited by the friction of two metallic surfaces, and given off in a constant stream or flux in all directions, and without any signs of *diminution* or *exhaustion.* In reasoning on this subject, we must not forget *that most remarkable circumstance,* that the source of heat generated by friction in these experiments appears evidently to be *inexhaustible.* It is hardly necessary to

add that anything which any *insulated* body or system of bodies can continue to furnish *without limitation* cannot possibly be a *material substance;* and it appears to me to be extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated in those experiments, except it be motion." (Tyndall, *Heat a Mode of Motion,* pp. 41ff.)

9. I have already alluded to a theory of solar heat first enunciated and developed by Dr. Mayer, which, however, bold it may at first sight appear, deserves our serious attention. . . . Besides comets, planets and moons, a numerous class of smaller bodies belong to our system which, like the planets and comets, obey the law of gravitation, and revolve in elliptic orbits round the sun. It is they which, when they come within the earth's attraction, and are fired by friction, appear to us as meteors and falling stars.

It is easy to calculate both the maximum and the minimum velocity imparted by the sun's attraction to an asteroid circulating round him. The maximum is generated when the body approaches the sun from an infinite distance; the *entire pull* of the sun being then exerted upon it. The minimum is that velocity which would barely enable the body to revolve round the sun close to his surface. The final velocity of the former, just before striking the sun, would be 390 miles a second, that of the latter 276 miles a second. The asteroid on striking the sun with the former velocity, would develop more than 9,000 times the heat generated by the combustion of an equal asteroid of coal; while the shock in the latter case, would generate heat equal to that of the combustion of upwards of 4,000 such asteroids. It matters not, therefore, whether the substances falling into the sun be combustible or not; their being combustible would not add sensibly to the tremendous heat produced by their mechanical collision. Here, then we have an agency competent to restore his lost energy to the sun, and to maintain a temperature at his surface which transcends all terrestrial combustion. (*Ibid.,* p. 519.)

10. If we grasp a coin and a bit of paper between the thumb and finger, and release both at the same instant, the coin will reach the floor first. It would seem as if a heavy body falls faster than a light body. Galileo was the first to show the

falsity of this assumption. He let drop from an eminence iron balls of different weights: they all reached the ground at the same instant. Hence he concluded that *the velocity of a falling body is independent of its mass.* He also dropped balls of wax with the iron balls. The iron balls reached the ground first. Are some kinds of matter affected more strongly by gravitation than others? If a coin and several bits of paper are placed in a glass tube, the air exhausted, and the tube turned end for end, it will be found that the coin and the paper will fall with equal velocities. *Hence the earth attracts all matter alike.* A wax ball of the same size as an iron ball meets with the same resistance from the air that the iron ball does; but since the mass of the former is less than that of the latter, the force acting on the former is less, and a less force cannot overcome the same resistance so quickly; consequently in the air the wax balls fall a little more slowly. We conclude, therefore, that *in a vacuum all bodies fall with equal velocities.* (Gage, *Introduction to Physical Science,* p. 90.)

11. It was believed that, as the organic compounds are elaborated under the influence of the life process, there must be something about them which distinguishes them from the inorganic compounds in whose formation the life process has no part. Gradually, however, this idea has been abandoned; for, one by one, the compounds which are found in plants and animals have been made in the chemical laboratory, and without the aid of the life process. The first instance of the preparation of an organic compound by an artificial method was that of urea. This substance was obtained by Wohler in 1828 from ammonium cyanate. When a water solution of the latter is allowed to evaporate, urea is deposited. Up to the time of Wohler's discovery, the formation of urea, like that of other organic compounds, was thought to be intimately and necessarily connected with life; but it was thus shown that it could be formed without the intervention of life. Afterward it was shown that potassium cyanide can be made by passing nitrogen over a heated mixture of carbon and potassium carbonate; and, as potassium cyanate can be made from the cyanide by oxidation, it follows that urea can be made from the elements. Finally, in 1856, Berthelot succeeded in making potassium formate by passing carbon monoxide over heated potassium hydroxide; and in making

acetylene, a compound, the composition of which is represented by the formula C_2H_2, by passing electric sparks between electrodes of carbon in an atmosphere of hydrogen. Since that time, every year has witnessed the artificial preparation, by purely chemical means, of compounds of carbon which are found in the organs of plants and animals.

It hence appears that the formation of the compounds of carbon is not dependent upon the life process; that they are simply chemical compounds governed by the same laws which govern other chemical compounds; and the name, *Organic Chemistry*, signifying, as it does, that the compounds included under it are necessarily related to organisms, is misleading. Organic chemistry is nothing but the *Chemistry of the Compounds of Carbon.* It is not a science independent of inorganic chemistry, but is just as much a part of chemistry as the chemistry of the compounds of sodium, or of the compounds of silicon, etc. (Remsen, *Organic Chemistry*, p. 1.)

12. The view at present held in regard to the nature of homology is founded primarily upon the idea that carbon is quadrivalent. If carbon is quadrivalent, it of course follows that the compound, marsh gas, CH_4, is saturated; that is, the molecule cannot take up anything without losing hydrogen. In order, therefore, that we may get a compound containing two atoms of carbon in the molecule, some of the hydrogen must first be given up. With our present views, we cannot conceive of union taking place directly between the molecules CH_4 and CH_4, but we can conceive of union taking place between the molecules CH_3 and CH_3, to form a molecule C_2H_6, which in turn is saturated. . . . The evidence in favor of this view will be presented when the reactions are considered by means of which the hydrocarbons are made. The explanation offered, and now generally accepted, involves the idea that carbon atoms have the power of uniting with each other. And, as the explanation for the relation between the first and second members is, in principle, the same as for the relation between the second and third, the third and fourth, etc., it appears that this power of carbon atoms to unite with one another is very extensive. It is to the power which carbon possesses of forming homologous series, or to the power of the atoms of carbon to unite with each

other, that we owe the large number of compounds of this element. (*Ibid.*, p. 22.)

13. We have to show that heat of all kinds, whether coming from the hand, or hot water, or steam, or red-hot iron, or a flame, or the sun, or from any other source, can be measured in the same way, and that the quantity of each required to effect any given change, to melt a pound of ice, to boil a pound of water, or to warm the water from one temperature to another, is the same from whatever source the heat comes.

To find whether these effects depend on anything except the quantity of heat received—for instance, if they depend in any way on the temperature of the source of heat—suppose two experiments tried. In the first a certain quantity of heat (say the heat emitted by a candle while an inch of candle is consumed) is applied directly to melt ice. In the second the same quantity of heat is applied to a piece of iron at the freezing point so as to warm it, and then the heated iron is placed in ice so as to melt a certain quantity of ice, while the iron itself is cooled to its original temperature.

If the quantity of ice melted depends on the temperature of the source from whence the heat proceeds, or on any other circumstance than the quantity of the heat, the quantity melted will differ in the two cases; for in the first the heat comes directly from an exceedingly hot flame, and in the second the same quantity of heat comes from comparatively cool iron.

It is found by experiment that no such difference exists, and therefore heat, considered with respect to its power of warming things and changing their state, is a quantity strictly capable of measurement, and not subject to any variations in quality or in kind. (Maxwell, *Theory of Heat*, p. 56.)

14. The fact that when equal weights of quicksilver and water are mixed together the resulting temperature is not the mean of the temperatures of the ingredients was known to Boerhaave and Fahrenheit. Dr. Black, however, was the first to explain this phenomenon and many others by the doctrine which he established, that the effect of the same quantity of heat in raising the temperature of the body depends not only on the amount of matter in the body, but on the kind of matter of which it is formed. (*Ibid.*, p. 65.)

15. Irvine, who contributed greatly to establish the fact that

the quantity of heat which enters or leaves a body depends on its capacity for heat multiplied by the number of degrees through which its temperature rises or falls, went on to assume that the whole quantity of heat in a body is equal to its capacity multiplied by the total temperature of the body, reckoned from a point which he called the absolute zero. The truth of such an assumption could never be proved by experiment, and its falsehood is easily established by showing that the specific heat of most liquid and solid substances is different at different temperatures. (*Ibid.*, p. 66.)

16. Whenever the volume of the substance is, like that of water, less in the liquid than in the solid state, the effect of pressure on a vessel containing the substance partly in a liquid and partly in a solid state is to cause some of the solid portion to melt, and to lower the temperature of the whole to the melting point corresponding to the pressure. If, on the contrary, the volume of the substance is greater in the liquid than in the solid state, the effect of pressure is to solidify some of the liquid part, and to raise the temperature to the melting point corresponding to the pressure. . . . When two pieces of ice at the melting point are pressed together, the pressure causes melting to take place at the portions of the surface in contact. The water so formed escapes out of the way and the temperature is lowered. Hence as soon as the pressure diminishes the two parts are frozen together with ice at a temperature below 32 degrees. This phenomenon is called regelation.

It is well known that the temperature of the earth increases as we descend, so that at the bottom of a deep boring it is considerably hotter than at the surface. We shall see that, unless we suppose the present state of things to be of no great antiquity, this increase of temperature must go on to much greater depths than any of our borings. It is easy on this supposition to calculate at what depth the temperature would be equal to that at which most kinds of stone melt in our furnaces, and it has been sometimes asserted that at this depth we should find everything in a state of fusion. But we must recollect that at such depths there is an enormous pressure, and therefore rocks which in our furnaces would be melted at a certain temperature may remain solid even at much greater temperatures in the heart of the earth. (*Ibid.*, p. 178.)

17. We shall now consider the nature of radiation, whether of light or heat, and show why we believe that what is called radiant heat is the same thing as what is called light, only perceived by us through a different channel. The same radiation which when we become aware of it by the eye we call light, when we detect it by a thermometer or by the sensation of heat we call radiant heat.

In the first place, radiant heat agrees with light in always moving in straight lines through any uniform medium. It is not, therefore, propagated by diffusion, as in the case of the conduction of heat, where the heat always travels from hotter to colder parts of the medium in whatever direction this condition may lead it.

The medium through which radiant heat passes is not heated, if perfectly diathermanous, any more than a perfectly transparent medium through which light passes is rendered luminous. But if any impurity or defect of transparency causes the medium to become visible when light passes through it, it will also cause it to become hot when traversed by radiant heat.

In the next place, radiant heat is reflected from the polished surfaces of bodies according to the same laws as light. A concave mirror collects the rays of the sun into a brilliantly luminous focus. If these collected rays fall on a piece of wood, they will set it on fire. If the luminous rays are collected by means of a convex lens, similar heating effects are produced, showing that radiant heat is refracted when it passes from one transparent medium to another.

When light is refracted through a prism, so as to change its direction through a considerable angle of deviation, it is separated into a series of kinds of light which are easily distinguished from each other by their various colors. The radiant heat which is refracted through the prism is also spread out through a considerable angular range, which shows that it consists of radiations of various kinds. (*Ibid.*, p. 232f.)

18. We must next consider the reasons which induce us to regard light as depending on a particular kind of motion in the medium through which it is propagated. These reasons are principally derived from the phenomena of the interference of light. . . . There are various methods by which a beam of light from a small luminous object may be divided into two portions,

which, after traveling by slightly different paths, finally fall on a white screen. Where the two portions of light overlap each other on the screen, a series of long narrow stripes may be seen, alternately lighter and darker than the average brightness of the screen near them. . . . If we stop either of these portions of light into which the original beam was divided, the whole system of bands disappears, showing that they are due, not to either of the portions alone, but to both united.

If we now fix our attention on one of the dark bands, and then cut off one of the partial beams of light, we shall observe that instead of appearing darker it becomes actually brighter, and if we again allow the light to fall on the screen it becomes dark again. Hence it is possible to produce darkness by the addition of two portions of light. If light is a substance, there cannot be another substance which when added to it shall produce darkness. We are therefore compelled to admit that light is not a substance.

Now is there any other instance in which the addition of two apparently similar things diminishes the result? We know by experiments with musical instruments that a combination of two sounds may produce less audible effect than either separately, and it can be shown that this takes place when the one is half a wave-length in advance of the other. Here the mutual annihilation of the sounds arises from the fact that a motion of the air towards the ear is the exact opposite of a motion away from the ear, and if the two instruments are so arranged that the motions which they tend to produce in the air near the ear are in opposite directions and of equal magnitude, the result will be no motion at all. Now there is nothing absurd in one motion being the exact opposite of another, though the supposition that one substance is the exact opposite of another substance . . . is an absurdity. (*Ibid.*, p. 234f.)

19. When a system of bodies at different temperatures is left to itself, the transfer of heat which takes place always has the effect of rendering the temperatures of the bodies more nearly equal. . . . Let us consider a number of bodies, all at the same temperature, placed in a chamber the walls of which are maintained at that temperature, and through which no heat can pass by radiation. No change of temperature will occur in any of these bodies. They will be in thermal equilibrium with

each other and with the walls of the chamber. . . . Now if any one of these bodies had been taken out of the chamber and placed among colder bodies there would be a transfer of heat by radiation from the hot body to the colder ones; or if a colder body had been introduced into the chamber it would immediately begin to receive heat by radiation from the hotter bodies round it. But the cold body has no power of acting directly on the hot bodies at a distance, so as to cause them to begin to emit radiations, nor has the hot chamber any power to stop the radiation of any one of the hot bodies within it. We therefore conclude with Prevost that a hot body is always emitting radiations, even when no colder body is there to receive them, and that the reason why there is no change of temperature when a body is placed in a chamber of the same temperature is that it receives from the radiation of the walls of the chamber exactly as much heat as it loses by radiation towards these walls.

If this is the true explanation of the thermal equilibrium of radiation, it follows that if two bodies have the same temperature the radiation emitted by the first and absorbed by the second is equal in amount to the radiation emitted by the second and absorbed by the first during the same time. The higher the temperature of a body the greater its radiation is found to be, so that when the temperatures of the bodies are unequal the hotter bodies will emit more radiation than they receive from the colder bodies and therefore on the whole heat will be lost by the hotter and gained by the colder bodies till thermal equilibrium is attained. (*Ibid.*, p. 240ff.)

20. By a series of beautiful experiments Aitken has shown that when ordinary moist air is cooled so as to form fog, each aqueous spherule founds itself upon a minute particle of foreign matter suspended as dust. When the air is nearly freed from dust by filtration through cotton wool, or otherwise, expansion produces a fine rain, consisting of comparatively few spheres of large diameter. The passage of a single electric spark between platinum points is sufficient to recharge the air with nuclei, so that on repeating the experiment the previous rain is replaced by a dense fog containing innumerable fine particles. (*Ibid.*, p. 293.)

21. Whether we consider the radiation of heat as effected by the projection of material caloric, or by the undulations of an

intervening medium, the outer surface of a hot body must be in a state of motion. . . . Now this motion is certainly invisible to us by any direct mode of observation, and therefore the mere fact of a body appearing to be at rest cannot be taken as a demonstration that its parts may not be in a state of motion. . . . Every hot body, therefore is in motion. We have next to enquire into the nature of this motion. It is evidently not a motion of the whole body in one direction, for however small we make the body by mechanical processes, each visible particle remains apparently in the same place, however hot it is. The motion which we call heat must therefore be a motion of parts too small to be observed separately; the motions of different parts at the same instant must, at least in solid bodies, be such that, however fast it moves, it never reaches a sensible distance from the point from which it started. . . . The molecules of all bodies are in a state of continual agitation. The hotter a body is, the more violently are its molecules agitated. In solid bodies, a molecule, though in continual motion, never gets beyond a certain very small distance from its original position in the body. The path which it describes is confined within a very small region of space. In fluids, on the other hand, there is no such restriction to the excursions of a molecule. . . . Hence in fluids the path of a molecule is not confined within a limited region, as in the case of solids, but may penetrate to any part of the space occupied by the fluid. (*Ibid.*, p. 310ff.)

22. To account for the propagation of heat and light—that is of radiant energy—we have postulated the existence of a medium filling all space. But the transference of the energy of radiant heat and light is not the only evidence we have in favor of the existence of an ether. Electric, magnetic, and electro-magnetic phenomena (and gravitation itself) point in the same direction.

It is a matter of common observation that attractions and repulsions take place between electrified bodies, magnets, and circuits conveying electric currents. Large masses may be set in motion in this way and acquire kinetic energy. If an electric current be started in any circuit, corresponding *induced* currents spring up in all neighboring conductors; yet there is no visible connection between the circuit and the conductors. To originate a current in any conductor requires the expenditure of energy.

How then is the energy propagated from the circuit to the conductors? If we believe in the continuity of the propagation of energy—that is, if we believe that when it disappears at one place and reappears at another, it must have passed through the intervening space, and therefore have existed there somehow in the meantime—we are forced to postulate a vehicle for its conveyance from place to place, and this vehicle is the ether. . . . Formerly an electrified body was supposed to have something called electricity residing upon it which caused the electrical phenomena, and an electric current was regarded as a flow of electricity travelling along the wire, while the energy which appeared at any part of the circuit (if considered at all) was supposed to have been conveyed along the wire by the current. The action of induction, however, and electromagnetic actions between bodies situated at a distance from each other, lead us to look upon the medium surrounding the conductors as playing a very important part in the development of the phenomena. It is, in fact, the storehouse of the energy. . . . When we speak of the charge of an electrified conductor we refer to the charge of energy in the ether around it, and when we talk of the electric flow or current in a circuit we refer to the only flow we know of, viz. the flow of energy through the electric field into the wire. (Preston, *The Theory of Light*, p. 542f.)

23. But who knows if the solar system and all the visible stars are not altogether moving off through space at the rate of a mile or a thousand miles a second? How can we tell unless we have something that is still and fixed to measure the motion by?

It seemed until recently that we had such a fixture, the ether. We know of the sun and stars only from the light that comes from them to us. Light, as we can prove by simple experiments, consists of wave motion. Now can you have wave motion without something to wave? Sound waves are conveyed by air; but there is no air between the earth and the sun. So as nothing could be found to fill this empty space scientists had to invent something to satisfy their sense of the fitness of things. The ether was the product of their excogitations. It was a British invention, devised in the Royal Institution, whence have come so many useful theories and discoveries.

The ether, as Salisbury said, is simply the nominative of the verb "to undulate." (Slosson, *Easy Lessons in Einstein*, p. 8.)

24. Four successive methods of explanation have been used to account for gravitation. First it was assumed by the ancient Babylonians and Hebrews that the sun and stars were living beings, gods or angels, moving of their own volition around the earth, or at least that each was guided in its orbit by a particular god or angel. The later Greeks of Ptolemy's time supposed the heavenly bodies to be set in concentric crystal spheres and so revolved; I presume by somebody turning a crank behind the scenes. Then came Newton and said, "Let's discard the Ptolemaic spheres and all mechanical connection and assume a *force* of gravitation attracting all bodies in proportion to their masses and inversely proportional to the square of the distances separating them." Now comes Einstein and says, "Let's discard this hypothetical force and simply assume that the field of time and space traversed by a moving body is altered if there is another body in the vicinity." (*Ibid.*, p. 76f.)

25. *Mutable Theories and Stable Facts.*—There is a feeling very prevalent among the general public interested in such things that the foundations of modern science are being swept away by the recent discoveries. The layman has been led to believe that such laws as gravitation, the conservation of matter and the immutability of the elements are the most certain and absolute truths of science. But now he hears respectable men of science talk calmly about the decay of matter and the transformation of one element into another, and gravely consider a theory which makes invalid Newton's three laws of motion. It surprises, even shocks, him, as much as it would to have a convention of bishops discuss the question of whether there is a God, or the Supreme Court agree to set aside the Constitution of the United States, or a congress of physicians resolve that all medicine does more harm than good. He knows that the mere broaching of such heretical views in these assemblies would be met with a storm of indignation and that all the weapons of contempt, ridicule and even personal spite, would be directed against the rash innovator. Therefore he is astonished and puzzled to see that in the scientific world these revolutionary theories are received with interest and even pleasure, and in the criticism to which they are subjected there is scarcely a trace of animosity. And he does not see why men of science who have accepted doctrines apparently contradictory to their

former teachings do not appear shamefaced and apologetic before the public, like augurs whose tricks had been exposed.

The difficulty of the layman arises from his not understanding how a scientist looks at his science; not realizing how firmly he holds to its facts and how loosely he holds to its theories. The scientist never bothers his head with the question whether a particular theory is true or false. He considers it simply as more or less useful, more or less adequate, succinct and comprehensive. A theory is merely a tool, and he drops one theory and picks up another at will and without a thought of inconsistency, just as a carpenter drops his saw and picks up his chisel. He will say that the earth moves around the sun one moment, and the next will revert to the theory of Chaldean astronomers, because it is more convenient, and say, "the sun rises."

Really, the new discoveries are not so upsetting to science as they appear to the general public. Unexpected and revolutionary as they are, no page of millions that record the experiments and observations of science is invalidated. No man's work is proved wrong. Revolutions in science do not destroy; they extend. (*Ibid.*, p. 99f.)

26. If we consider the simplest known atom, namely that of Hydrogen, we can make various hypotheses somewhat as follows:

(1) The main bulk of the atom may consist of ordinary matter (whatever unknown entity is hidden by that familiar phrase), associated with sufficient positive electricity (whatever that may be) to neutralize the charge belonging to the electron or electrons which undoubtedly exist in connection with each atom.

(2) Or the bulk of the atom may consist of a multitude of positive and negative electrons, interleaved, as it were, and holding themselves together in a cluster by their mutual attractions, either in a state of intricate orbital motion, or in some static geometrical configuration, kept permanent by appropriate connexions.

(3) Or the bulk of the atom may be composed of an indivisible unit of positive electricity, constituting a presumably spherical mass or "jelly," in the midst of which an electrically equivalent number of point electrons are as it were "sown"; these electrons

probably distributing themselves in rings, after the fashion of Alfred Mayer's floating magnetic needles, and revolving in regular orbits about the center of the jelly with a force directed to that center, and varying as the direct distance from it.

This hypothesis, in spite of obvious weaknesses connected with the nature of the positive unit, has great attractiveness:—for it explains the constant period of an orbit; it can explain the occurrence of visible radiation by perturbations of the orbit during collision; and it has been shown by J. J. Thompson to be capable of carrying us a long way towards a rational electrical theory of Mendeleef's series of the chemical elements, together with some of their chemical—especially their electrochemical—properties, and some features of their spectra. Moreover it goes further and explains in a fairly natural manner, and without artificiality, the gradual degradation of atomic energy by slow uncompensated and unperceived radiation; the consequent gradual oncoming of instability; and the occasional cataclysmic transmutation of one element into another, or rather into others, with explosive violence,—as observed in the facts of radioactivity. It gives in fact a rational though preliminary view of the hypothetical evolution of all matter, which many known circumstances now tend to support; and it accounts in a kinetic fashion for the immense store of intra-atomic energy. Nevertheless it is very far from being an established theory, and another view that can be taken of the rest of the atom is:

(4) That it consists of a kind of interlocked admixture of positive and negative electricity, indivisible and inseparable into units, and incapable of being appreciably sheared by applied forces, but incorporated together as a continuous mass; in the midst of which one or more isolated individualized electrons may move about and carry on that display of external activity which confers upon the atom its observed properties.

(5) A fifth view of the atom would regard it as a central "sun," of extremely concentrated positive electricity at the center. With a multitude of electrons revolving in astronomical orbits, like asteroids, within its range of attraction. But this would give a law of inverse square for the force, and consequently periodic times dependent on distance, which appears not to correspond with anything satisfactorily observed.

All these views however are painfully indefinite, except the third one, which regards positive electricity as an indivisible unit of perfectly unresisting uniform material (though "material" is not the right word), of spherical form the size of an atom, in the midst of which a definite geometrical arrangement of electrons are revolving with known frequency in specified groups or rings. . . . This hypothesis enables mechanical laws and calculations to be applied with considerable fulness to the elucidation of the phenomena that would be displayed by such a "model" or hypothetical combination. And if the calculated phenomena are found to correspond with fact, it assuredly lends some strength to the hypothetical basis on which the calculation is founded; although it is certain to have to be modified somewhat in the light of growing experience as discovery proceeds. (Lodge, *Electrons*, p. 148ff.)

27. There are two parties to every observation—the observed and the observer. What we see depends not only on the object looked at, but on our own circumstances—position, motion, or mere personal idiosyncrasies. Sometimes by instinctive habit, sometimes by design, we attempt to eliminate our own share in the observation, and so form a general picture of the world outside us, which shall be common to all observers. A small speck on the horizon of the sea is interpreted as a giant steamer. From the window of our railway carriage we see a cow glide past at fifty miles an hour, and remark that the creature is enjoying a rest. We see the starry heavens revolve round the earth, but decide that it is really the earth that is revolving, and so picture the state of the universe in a way which would be acceptable to an astronomer on any other planet. (Eddington, *Space, Time, and Gravitation*, p. 30.)

28. The ether has ceased to take any very active part in physical theory and has, as it were, gone into reserve. A modern writer on electromagnetic theory will generally start with the postulate of an ether pervading all space; he will then explain that at any point in it there is an electromagnetic vector whose intensity can be measured; henceforth his sole dealings are with this vector, and probably nothing more will be heard of the ether itself. In a vague way it is supposed that this vector represents some condition of the ether, and we need not dispute that without some such background the vector

would scarcely be intelligible—but the ether is now only a background and not an active participant in the theory.

There is accordingly no reason to transfer to this vague background of ether the properties of a material ocean. Its properties must be determined by experiment, not by analogy. In particular there is no reason to suppose that it can partition out space in a definite way, as a material ocean would do. . . . Natural geometry depends on laws of matter; therefore it need not apply to the ether. Permanent identity of particles is a property of matter, which Lord Kelvin sought to explain in his vortex-ring hypothesis. This abandoned hypothesis at least teaches us that permanence should not be regarded as axiomatic, but may be the result of elaborate constitution. There need not be anything corresponding to permanent identity in the constituent portions of the ether; we cannot lay our finger at one spot and say "this piece of ether was a few seconds ago over there." Without any continuity or identity of the ether, motion through the ether becomes meaningless; and it seems likely that this is the true reason why no experiment ever reveals it. (*Ibid.*, p. 39f.)

29. Force, as known to us observationally, is like the other quantities of physics a relation. The force measured with a spring-balance, for example, depends on the acceleration of the observer holding the balance; and the term may . . . have no exact counterpart in a description of nature independent of the observer. Newton's view assumes that there is such a counterpart, an active cause in nature which is identical with the force perceived by his standard unaccelerated observer. Although any other observer perceives this force with additions of his own, it is implied that the original force in nature and the observer's additions can in some way be separated without ambiguity. There is no experimental foundation for this separation, and the relativity view is that a field of force can . . . be nothing but a link between nature and the observer. There is, of course, something at the far end of the link. . . . We shall have to study the nature of this unknown whose relation appears to us as force. Meanwhile we shall realize that the alteration of perception of force by non-uniform motion of the observer . . . is what might be expected from the nature of these quantities as relations only. (*Ibid.*, p. 43.)

30. Whilst the evidence for the electrical theory of matter is not so conclusive as at one time appeared, the theory may be accepted without serious misgivings. To postulate two entities, matter and electric charges, when one will suffice, is an arbitrary hypothesis, unjustifiable in our present state of knowledge. The great contribution of the electrical theory to this subject is a precise explanation of the property of inertia. It was shown theoretically by J. J. Thompson that if a charged conductor is to be moved or stopped, additional effort will be necessary simply on account of the charge. The conductor has to carry its electric field with it, and force is needed to set the field moving. This property is called inertia, and it is measured by *mass*. If, keeping the charge constant, the size of the conductor is diminished, this inertia increases. Since the smallest separable particles of matter are found by experiment to be very minute and to carry charges, the suggestion arises that these charges may be responsible for the whole of the inertia detected in matter. The explanation is sufficient; and there seems no reason to doubt that all inertia is of this electrical kind. (*Ibid.*, p. 61.)

31. It is quite possible that there might be a super-observer, whose views have a natural right to be regarded as the truest, or at least the simplest. A society of learned fishes would probably agree that phenomena were best described from the point of view of a fish at rest in the ocean. But relativity mechanics finds that there is no evidence that the circumstances of any observer can be such as to make his views preëminent. All are on an equality. Consider an observer A in a projectile falling freely to the earth, and an observer B in space out of range of any gravitational attraction. Neither A nor B feels any field of force in his neighborhood. Yet in Newtonian mechanics an artificial distinction is drawn between their circumstances; B is in no field of force at all, but A is really in a field of force, only its effects are neutralized by his acceleration. But what is this acceleration of A? Primarily it is an acceleration relative to the earth, but then that can equally well be described as an acceleration of the earth relative to A, and it is not fair to regard it as something located with A. Its importance in Newtonian philosophy is that it is an acceleration relative to what we have called the super-observer. This

potentate has drawn planes and lines partitioning space, as space appears to him. I fear that the time has come for his abdication. (*Ibid.*, p. 68.)

32. Was there any reason to feel dissatisfied with Newton's law of gravitation? Observationally it had been subjected to the most stringent tests, and had come to be regarded as the perfect model of an exact law of nature. The cases where a specific failure could be alleged were almost insignificant. There are certain unexplained irregularities in the moon's motion; but astronomers generally looked—and must still look—in other directions for the cause of these discrepancies. One failure only had led to a serious questioning of the law; this was the discordance of motion of the perihelion of Mercury. How small was this discrepancy may be judged from the fact that, to meet it, it was proposed to amend *square* of the distance to the 2.00000016 power of the distance. Further, it seemed possible though unlikely, that the matter causing the zodiacal light might be of sufficient mass to be responsible for this effect.

The most serious objection against the Newtonian law as an exact law was that it had become ambiguous. The law refers to the product of the masses of the two bodies; but the mass depends on the velocity—a fact unknown in Newton's day. Are we to take the variable mass, or the mass reduced to rest? Perhaps a learned judge, interpreting Newton's statement like a last will and testament, could give a decision; but that is scarcely the way to settle a point in scientific theory.

Further *distance,* also referred to in the law, is something relative to the observer. Are we to take the observer travelling with the sun or with the other body concerned, or at rest in the ether or in some other gravitational medium?

Finally is the force of gravitation propagated instantaneously, or with the velocity of light, or some other velocity? (*Ibid.*, p. 93.)

33. An unexplained advance of the orbit of Mercury had long been known. It had occupied the attention of Le Verrier, who, having successfully predicted the planet Neptune from the disturbances of Uranus, thought that the anomalous motion of Mercury might be due to an interior planet, which was called Vulcan in anticipation. But, though thoroughly sought for, Vulcan has never turned up. Shortly before Einstein

arrived at his law of gravitation, the accepted figures were as follows. The actual observed advance of the orbit was 574″ per century; the calculated perturbations produced by all the known planets amounted to 532″ per century. The excess of 42″ per century remained to be explained. Although the amount could scarcely be relied on to a second of arc, it was at least thirty times as great as the probable accidental error.

The discrepancy from the Newtonian gravitational theory is in agreement with Einstein's prediction of an advance of 43″ per century. . . . Unfortunately it is not possible to obtain any further test of Einstein's law of gravitation from the remaining planets. We have to pass over Venus and the Earth, whose orbits are too nearly circular to show the advance of the apses observationally. Coming next to Mars with a moderately eccentric orbit, the speed is very much smaller, and the predicted advance is only 1″.3 per century. Now the accepted figures show an observed advance (additional to that produced by known causes) of 5″ per century, so that Einstein's correction improves the accordance of observation with theory; but, since the result for Mars is in any case scarcely trustworthy to 5″ owing to the inevitable errors of observation, the improvement is not very important. The main conclusion is that Einstein's theory brings Mercury into line, without upsetting the good accordance of all the other planets. (*Ibid.*, p. 125.)

34. There are two types of color-blindness which are common, both of which are characterized by certain confusions of reds and greens. If two individuals, one belonging to each type, be given assortments of colored worsteds to arrange according to their color resemblances, the arrangements made by both men will be erroneous to the normal eye, and the arrangement made by the one man will appear erroneous to the other. From the three-color hypothesis, it may be deduced that one arrangement will appear correct to the normal eye when it is illuminated by daylight from which only a band of yellowish green has been removed: and that the other arrangement will appear correct if illuminated by daylight from which only the red has been removed. The *experiment* then consists in arranging the illumination as described, and the results being found to agree with the prediction from the hypothesis, the hypothesis is so far (but so far only) confirmed.

(Dunlap, *Mysticism, Freudianism, and Scientific Psychology,* page 114.)

35. The Freudian doctrine of consciousness as a *stuff* which, after it has functioned, is stored away somewhere like the printer's type which is returned to its case after being used, has no more empirical basis than has an exactly corresponding conception of finger movements which, after having occurred, are somewhere stored up as motionless movements. Just as the movement exists only during the motion, so consciousness exists only while one is conscious: and just as the original occurrence of the movement leaves biological structures so modified that the movement may occur again, so consciousness occurring once, leaves the biological mechanism so modified that it may recur. (*Ibid.,* p. 124.)

36. The believer in "unconscious mental processes" . . . might state his claim as follows: In addition to mental processes, i.e., organic processes involving consciousness (awareness), there are other processes, which while they do not involve awareness, involve something which is more than mere physiological process: something resembling consciousness, but not conscious. The "unconscious mental" factor is therefore an x, an unknown, and cannot be pointed out in any definite experience. Such an hypothesis might be made. One might also hypothesize a y factor, a z factor, and an infinity of other factors, all equally unknown, equally beyond experience. But science does not indulge in the positing of hypothetical entities whose only qualification is that, they being unknown, we cannot know that they do not exist. Hypotheses which are by their nature removed from any possibility of verification, are never constructed in science. (*Ibid.,* p. 126.)

37. So far as the empirical evidence goes, it is altogether in favor of the reaction-arc hypothesis. Perception, at least, is impossible if the arc is interrupted. In the case of vision, experiments on animals, and human clinical cases, make this point clear. Destroying the retinæ; cutting the optic nerves; cutting the optic tract behind the brain stem; destroying the occipital lobes of the cerebral hemispheres to which the optic tracts lead; or cutting the connections between the occipital lobes and the remainder of the cerebral hemispheres; produce one and the same result—blindness—by interrupting the arcs

from the visual receptors to the effector systems, and destroying the possibility of a visual reaction. (*Ibid.*, p. 135.)

38. What goes on in the neuron when it acts has not been definitely determined. Theories have varied at different periods from assuming that some fluid was transmitted through the nerves or that some wave was propagated along the substance of the nerve, to the assumption that the action was electrical in character. To-day a generally accepted theory is that the impulse is due to some form of chemical change which spreads through the neuron. This hypothesis is supported by several facts. (1) The rate of propagation about one to two hundred metres a second, is altogether too slow for electrical transmission, but is within the limits of chemical action. (2) The action of the nerve-cell is accompanied by electrical phenomena. Whenever a nerve is stimulated, an electric current passes from the cut end of the nerve to the uninjured sheath. If a frog's nerve be dissected out and one end be connected to a point on the neurilemma through a delicate galvanometer, the galvanometer will indicate the passage of an electric current when the nerve is stimulated in any way. Similar electric currents are induced, as is well known, by the chemical action in a battery. (3) The action of nerve cells frees certain chemical substances, products of decomposition, that are removed by the blood. These three facts point to the assumption that action of the nerves is due to some chemical change. This assumption fits in with most of our detailed knowledge of the action of the nervous system. (Pillsbury, *Essentials of Psychology,* p. 28.)

39. Two explanations have been given of the origin of instincts. The simpler is that instincts are merely inherited habits. On this theory some ancestor learned a movement, and the habit was transmitted to his descendants and became a racial possession. Were the biologist willing to accept this theory, the explanation of the origin of instincts would be very simple. Unfortunately the evidence that a change wrought in one individual is transmitted to his offspring is not accepted by the great majority of biologists. Weismann has demonstrated to the satisfaction of many of his colleagues that the structures of the body are so completely set off from the tissues which are to continue the race that the changes in the body have no influence upon the inheritance of the offspring. The cells from

which the progeny are to develop have in potentiality at the birth of the individual all the characteristics,—they are influenced only by the factors that weaken or destroy the body as a whole. Whatever be the outcome of the biological controversy, it is necessary for the psychologist to construct a theory of instinct on the assumption of the accepted biological theory.

On this theory of Weismann, instincts come not through a change in the habits of the individual, but through some chance change in the characteristics of the germ plasm. It is a fact that, while the characteristics of the parent are transmitted, they are not transmitted accurately, there is always variation in the characters. If one sows a thousand seeds from the same plant, the young plants will show a wide range of variation from the parent plant and from each other. The theory of the development of the instincts assumes this same tendency to variation in the nervous system and in the instincts that correspond to the nervous connections. If this known fact of variation be accepted, all that is necessary for the development of an instinct is that some selection be made from the variations. This selecting agent has been found by all the evolutionary theories in the environment. When a variation in responses better suited to the environment than the older responses makes its appearance, the animal that shows the variation will be more likely to survive. If this variation is inherited, as it tends to be, the offspring of this animal will survive in greater numbers and in time will outnumber those with less adequate responses. In brief, variations in responses are constantly appearing as the result of changes in the structure of the germ plasm. The animal that has the more beneficial responses will live and its offspring will increase, while any animal that develops variations unsuited to the environment will be destroyed, or the offspring will be fewer. As a result of this variation in structure and response, with selection of the animals which show suitable variations, instincts become constantly more suited to the conditions of life, and also become more and more complicated. Variation and selection can account for any instinct, granted only a sufficiently long time for the variation to develop. (*Ibid.*, p. 270f.

40. *Mental and Physical Fatigue One.*— . . . It is generally

believed that one may rest from mental work while exercising, but experiments indicate that capacity for mental work is decreased by physical work if it is too difficult. If one takes a vigorous run or other severe exercise between two periods of the same sort of work, . . . the capacity for mental work is diminished rather than increased. Here as before the effect will depend upon the severity of the task. If the exercise be mild, one will rest relatively, just as one does during less difficult mental work. . . . The identity of mental and physical fatigue has been demonstrated many times, both that mental work induces physical fatigue and that physical work induces mental fatigue. One cannot do severe mental work effectively after a hard day of physical labor, and experiments show that one is less capable of physical after hard mental work. (*Ibid.*, p. 364f.)

41. There is a type of pain which modifies muscular action in a curious way. We have already stated that local pain serves an adaptive purpose. In this light let us now consider headache. Headache is one of the commonest initiatory symptoms of the various infections, especially of those infections which are accompanied by no local pain and by no local muscular action. . . . In the diseases in which local pain is absent such as the exanthemata, typhoid fever, and auto-intoxication, which have no dominating local disturbances to act as policemen to put the individual to bed and to make him refuse food that he may be in the most favorable position to combat the oncoming disease, in such cases in which these masterful and beneficent local influences are absent we postulate that headache has been evolved to perform this important service.

On the hypothesis that it is good for the individual who is acutely stricken by a disease or who is poisoned by auto-intoxication to rest and fast, and that the muscular system obeys the imperial command of pain, and in view of the fact that the brain is not only in constant touch with the conditions of every part of the body but that it is also the controlling organ of the body, one would expect that in these diseases the major pain whose purpose it is to govern general muscular action would be located in the head and there we find it. How curious and yet how intelligible is the fact that, though a headache may be induced by even a slight auto-intoxication, an abscess

may exist within the brain without causing pain. When an obliterative endarteritis is threatening a leg with anemic gangrene, or when one lies too long in the same position on a hard bed, there is threatening injury from local anemia, and as a result there is acute pain, but when the obliterative endarteritis threatens anemia of the brain, or when an embolism or thrombosis has produced anemia of the brain, there may be no accompanying pain. The probable explanation of the pain which results in the first instance and the lack of pain in the second is that in the former muscular action constitutes a self-protective response, but in the other it does not. Diseases and injuries of the brain are notoriously difficult to diagnosticate. This may well be because it has always been so well protected by the skull that there have been evolved within it few tell-tale self-protective responses, so that in the presence of injury within itself the brain remains remarkably silent. It should occasion no surprise that there are in the brain no receptors, the mechanical stimulation of which can cause pain, because its bony covering has always prevented the adaptive implantation within it of contact pain receptors. (Crile, *The Origin and Nature of the Emotions,* pp. 80ff.)

42. A convincing proof that environment has been the creator of man is seen in the absolute adaptation of the nociceptors as manifested in their specific response to adequate stimuli, and in their presence in only those parts of the body which throughout the history of the race have been most exposed to harmful contacts. We find they are most numerous in the face, the neck, the abdomen, the hands, and the feet; while in the back they are few in number, and within the bony cavities they are lacking.

Instances of the specific responses made by the nociceptors might be multiplied indefinitely. Sneezing, for example, is a specific response made by the motor mechanism to stimulation of nociceptors in the nose. While stimulation of the larynx does not produce a sneeze but a cough; stimulation of the nociceptors of the stomach does not produce cough but vomiting; stimulation of the nociceptors of the intestine does not produce vomiting, but increased peristaltic action. There are no nociceptors misplaced; none wasted; none that do not make an adequate response to adequate stimulation.

Another most significant proof that the environment of the past has been the creator of the man of to-day is seen in the fact that man has added to his environment certain factors to which adaptation has not as yet been made. For example, heat is a stimulus which has existed since the days of prehistoric man, while the x-ray is a discovery of to-day; to heat, the nociceptors produce an adequate response; to the x-ray there is no response. There was no weapon in the prehistoric ages which could move at the speed of a bullet from a modern rifle, therefore, while slow penetration of the tissues produces great pain and muscular response, there is no response to the swiftly moving bullet. . . . Powerful response is made to crushing injury by environmental forces; to such injuring contacts as resemble the impacts of fighting; to such tearing injuries as resemble those made by teeth and claws. On the other hand, the sharp division of tissue by cutting produces no adequate response; indeed, one might imagine that the body could be cut to pieces by a superlatively sharp knife applied at tremendous speed without material adaptive response.

These examples indicate how the history of the phylogenetic experiences of the human race may be learned by a study of the position and the action of the nociceptors, just as truly as the study of the arrangement and variations in the strata of the earth's crust discloses to us geologic history. (*Ibid.*, pp. 130f.)

43. Discuss the principle involved in the following experiment: "Can you tell the difference in weight between these two shells?" asked Edwin. "Yes, I can: this one is the lighter of the two." "Tell me now which one is the lighter of the two," Edwin asked again. "I can't do it; they are both the same." "It is strange that you could tell the difference the first time and not the second, although I simply added, for the second test, the same weight of small lead shots to each shell used in the first set." (Snow, *Problems in Psychology*, p. 26.)

44. Discuss the principle involved in the fact stated below: In a demonstration in geometry the figure represents all objects of similar form, without reference to their size or the materials with which they are made. The demonstration will be applicable to pieces of paper or tracts of land, just as truly as to the figure drawn in chalk or pencil. (*Ibid.*, p. 76.)

45. Someone has thrown a stone through my window pane. Can you not see the glass scattered all over the floor of the room? But I cannot find the stone. It must have been the wind that broke the glass. (*Ibid.*, p. 78.)

46. One of the two best-known theories of color-vision starts out from the facts designated by the term "three physical primaries." . . . A practical demonstration of the three "physical primaries is afforded by examining a Lumiere "autochrome" plate, that is, a transparency, made by the Lumiere process of three-color-photography. We are not concerned here with the technique of the photographic process itself but only with the finished product. If such an autochrome be viewed in transmitted light, by holding it against the brightly illuminated sky, or better still, by placing one of lantern-slide size in the carrier of a stereopticon, the objects photographed will appear in their natural colors. At least this effect is approximated to a remarkable degree. In this instance all our color-sensations, including those of white and gray, are due to the combinations in different proportions of red, green, and blue-violet lights. What adds to the interest of this demonstration, is the ingenious method of effecting the physical mixture of these kinds of light. As substantially the same method of color-mixture is also used for other artistic purposes, as for instance, in tapestry and that form of painting which is known as "pointilistic," it shall be briefly explained here.

Lumiere autochrome plates are provided with a layer of minute starch grains, dyed red, green, and blue-violet, and in the proportion of four green to three red and two blue. On every inch of the plate there are about four millions of these grains, not superimposed upon one another, but in juxtaposition. When the photograph is taken, light must first pass through this "color-filter," before it reaches the sensitive film, and when the finished picture is projected on a screen by means of a stereopticon, light is similarly filtered, before it reaches the screen. Hence only red, green, and blue-violet lights reach the eye, when we look at such a screen.

An observer near the screen, say about two or three feet away from it, can see the magnified images of the starch grains clearly. Those who are about ten feet away from the screen can no longer distinguish the juxtaposed colored

spots. They see instead the objects photographed in their natural colors.

The reason for this is that the human eye is unable to focus such small objects as these colored spots separately, when they are viewed beyond a certain distance. It is as if we tried to read print at such a distance: all letters would be blurred. In technical language this is expressed by saying that the juxtaposed colored spots on the screen cause "circles of diffusion" on the retina; that is, they stimulate each a wider area of the retina than is consistent with their distinct vision. The circles of diffusion, caused by neighboring colored spots on the screen must necessarily overlap on the retina. Now this means that the same retinal elements are stimulated simultaneously by different colored lights. It is owing to the ingenious method, employed in making these photographic transparencies, that the three "primary" lights, red, green, and violet-blue, reach the retina in different proportions, according as we look at one portion of the screen or another. Hence it is that a white object, thus photographed, looks white, a golden one golden, and so forth. (Gruender, *Experimental Psychology*, pp. 89f.)

47. Fill a small box with shot. Then take a larger box of the same shape and material, and place enough shot in it that the two boxes are of exactly the same weight. This determination should be made by means of an accurate balance. Now ask someone, who does not know how these boxes were prepared, to lift one after the other and to tell which of the two seems the heavier. It will invariably be found that the smaller box is judged to be decidedly the heavier one. In lifting the small box we experience a certain effort. Seeing the other box of the same material and shape, but larger than the first, we expect to experience more of an effort in trying to raise it. The actual effort falls considerably below our expectation, and as a result we judge the larger box to be the lighter one.

The result will in most cases be the same, if the small box is actually somewhat lighter than the larger one and even after the subject of the experiment has found out how the boxes were prepared; so strong is the influence of suggestion.

That it is really the expectation which is responsible for the illusion can be tested in the following manner. Prepare two additional boxes, larger than the first two, and of equal weight,

shape, and size. Enclose the small box in one of the latter, the larger one in the other. Then no expectation influences us in testing the weights and the illusion is gone. There may indeed be errors, but just as well in favor of the one as the other box. (*Ibid.*, pp. 231ff.)

48. In a quiet room have an assistant hold a watch at such a distance from your ear that you can just hear its ticking. When this distance has been ascertained, close the eyes so as to exclude all disturbing visual stimuli, and being seated comfortably, listen as attentively as possible to the ticking of the watch. The faint sound you hear is near the threshold of audibility and thus very favorable for the observation of fluctuations of attention. It would be best to employ a continuous sound of medium pitch and just as soft as the ticking of the watch. But this is difficult. Though the ticking of the watch is not, strictly speaking, an unchanging object of attention, the succession of faint sounds, of which it consists, is monotonous and thus equivalently an unchanging object. It is easy, moreover, to prescind from all associations which the ticking might arouse; such associations, if there be any, are not liable to be very obtrusive, so that you can direct your attention exclusively to what is immediately given in your sensory experience.

These conditions secured, we find that the ticking of the watch will periodically grow still fainter and drop entirely out of our mind. There will be fluctuations of attention which can be recorded in the form of a curve. (*Ibid.*, p. 241.)

49. Halstead of the New Jersey Experiment Station reported a small fertilizer experiment with peas in which five varieties of standard trees were fertilized as follows: (1) No treatment; (2) 500 pounds per acre of a mixture of equal parts bone meal, acid phosphate, and muriate of potash; and (3) the same as (2) with the addition of 150 pounds of nitrate of soda per acre. It was found that the weight of fruit was greatest on the phosphorus and potash plat, second on the check plat, and lowest on the complete fertilizer plat. The average weight of individual fruits was largest on the complete fertilizer plat and smallest on the check plat. (Report of the New York State Experiment Station, 1920-21.)

50. At the Ohio Experiment Station, corn, oats, and wheat were each grown continuously on the same land on the one

hand, and in rotation with each other and with clover and timothy on the other for twenty-five years. During the last five years the yields of the rotated crops have been from two to three times as large as those from the crops grown continuously. (*Ibid.*)

51. In 1919 an experiment was made at the Illinois Experiment Station on the clipping of oats. In one plot the plants were cut at a height of 8 inches. There was a yield of 43 bushels per acre where clipped and of 44 bushels where not clipped. (*Ibid.*)

52. Experiments in taking and classifying nose-prints were begun in October, 1921. As with finger-prints, two important points must be considered. Is the cow's nose-print different from that of every other cow? And does the pattern remain the same at all ages? As with the human finger, both these questions must be answered in the affirmative before the nose-print will be of value in identification.

The prints of more than 350 animals have been taken and carefully scrutinized. So far no two have been found even sufficiently alike to cause any uncertainty as to their being from different animals. And both growing calves and older animals have been nose-printed for five consecutive months without indicating any change of design. A careful study of the prints indicates that while there is enlargement of the nose, the arrangement of the ridges remains fixed. (*Scientific American,* quoted by *Literary Digest,* Nov. 11, 1922.)

53. Taking two female rats of known age and strain, one with a cancerous condition and the other free from any pathological condition, Mr. Butts connected them with copper and platinum wires to a galvanometer. In the case of the cancerous rat, the one end of the wire was placed into the healthy border of the tumor.

The galvanometer was delicate enough to register a ten-billionth of a volt. When a contact key had been inserted in the circuit there was a deflection of the galvanometer mirror to the right. The cancerous rat acted as the positive pole and the normal rat the negative. By attaching the galvanometer to ordinary dry cells the demonstrator received verification of his theory. By reversing the electrodes in the rats, the deflection occurred in the opposite direction.

Possibility of interference of thermal changes and the degree of anesthesia was eliminated by the experiment.

"After repeating the experiment nearly 200 times," Mr. Butts said, "I felt I was justified in saying that tumor-bearing animals have a greater potential than normal rats."

Once that fact was established, Mr. Butts substituted a strong positive colloid of iron for the cancerous rat and a strong negative colloidal sulphur for the normal rat, and received the same result with the galvanometer. Philadelphia *Public Ledger.*)

54. It is quite clear that there cannot be life on the stars. Nothing solid or even liquid can exist in such furnaces as they are. Life exists only on planets, and even on these its possibilities are limited. . . . In considering the possibility of life as we know it we may at once rule out the more distant planets from the sun, Uranus and Neptune. They are probably too hot. We may also pass over the nearest planet to the sun, Mercury. We have reason to believe that it turns on its axis in the same period as it revolves round the sun, and it must therefore always present the same side to the sun. This means that the heat on the sunlit side of Mercury is above boiling-point, while the cold on the other side must be between two and three hundred degrees below freezing-point.

The planet Venus . . . is of nearly the same size as the earth, and it has a good atmosphere, but there are many astronomers who believe that, like Mercury, it always presents the same face to the sun, and it would therefore have the same disadvantage. . . .

We turn to Mars. . . . The late Professor Percival Lowell, who made a lifelong study of Mars, maintained that there are hundreds of straight lines drawn across the surface of the planet, and he claimed that they are beds of vegetation marking the sites of great channels or pipes by means of which the "Martians" draw water from their polar ocean. Professor W. H. Pickering, another high authority, thinks that the lines are long, narrow marshes fed by moist winds from the poles. There are certainly white polar caps on Mars. They seem to melt in the spring, and the dark fringe round them grows broader.

Other astronomers, however, say that they find no trace of

water-vapor in the atmosphere of Mars, and they think that the polar caps may be simply thin sheets of hoar-frost or frozen gas. They point out that, as the atmosphere of Mars is certainly scanty, and the distance from the sun is so great, it may be too cold for the fluid water to exist on the planet. . . .

Next to Mars, going outward from the sun is Jupiter. . . . But there is no life on Jupiter. The surface which we see in photographs is a mass of cloud or steam which always envelops the body of the planet. It is apparently red-hot. . . . Saturn is in the same interesting condition. . . . Mars and Venus are therefore the only planets, besides the earth, on which we may look for life; and in the case of Venus, the possibility is very faint. (*The Outline of Science,* edited by J. Arthur Thompson, Vol. I, p. 28f.)

55. A very interesting recent research shows that there are two kinds of red stars; some of them are among the oldest and some are among the youngest. The facts appear to be that when a star is first formed it is not very hot. It is an immense mass of diffuse gas glowing with a dull red heat. It contracts under the mutual gravitation of its particles, and as it does so it grows hotter. It acquires a yellowish tinge. As it continues to contract it grows hotter and hotter until its temperature reaches a maximum as a white star. At this point the contraction process does not stop, but the heating process does. Further contraction is now accompanied by cooling, and the star goes through its color changes again, but this time in the inverse order. It contracts and cools to yellow and finally to red. But when it again becomes a red star it is enormously denser and smaller than when it began as a red star. Consequently the red stars are divided into two classes called, appropriately, Giants and Dwarfs. (*Ibid.,* p. 38.)

56. This molecular movement can, in a measure, be made visible. It was noticed by a microscopist named Brown that, in a solution containing very fine suspended particles, the particles were in constant movement. Under a powerful microscope these particles are seen to be violently agitated; they are each independently darting hither and thither somewhat like a lot of billiard balls on a billiard table, colliding and bounding about in all directions. Thousands of times a second these encounters occur, and this lively commotion is always going

on, this incessant colliding of one molecule with another is the normal condition of affairs; not one of them is at rest. The reason for this has been worked out, and it is now known that these particles move about because they are being incessantly bombarded by the molecules of the liquid. The molecules cannot, of course, be seen, but the fact of their incessant movement is revealed to the eye by the behavior of the visible suspended particles. This incessant movement in the world of molecules is called the Brownian movement, and is a striking proof of the reality of molecular motions. (*Ibid.*, p. 248f.)

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CHAPTER XIII

FALLACIES

A textbook on logic may be described positively as a system of directions for correct thinking, or negatively as a collection of *warnings* for the avoidance of fallacy. It is the latter aspect which is to be presented in the present chapter. The purpose of the chapter is twofold: to review such erroneous modes of thinking as have been discussed in previous chapters; and to direct attention to the more important of the words and phrases which have been used as names of fallacious modes of reasoning. The chapter may then be considered as a sort of *dictionary of fallacy*. It is not to be supposed, however, that all of the "fallacies" which are enumerated can be clearly distinguished. Not infrequently two or more terms will be found to be designations of what is virtually one and the same error. Moreover, just as it is possible to "reduce" one mood of valid reasoning to another, so it is frequently possible to reduce one fallacy to another. Accordingly, while in the list now to be given each erroneous mode of reasoning—or rather, each *name* for an erroneous mode of reasoning—is assigned a number, as if all were distinct and equally important, it is not to be supposed that all are really co-ordinate. As a matter of fact it is clear that some are only special cases of others. In short, our list of "fallacies" is to be considered as a catalog of names rather than of things.

1. **Affirming the Consequent.**
2. **Denying the Antecedent.**

3. Imperfect Disjunction.

4. Incomplete Enumeration. (For this and the preceding see Chap. III.)

5. Invalid Conversion.—The most common errors in conversion are attempts to convert A and O propositions *simply* instead of by limitation or contraposition. (See Chap. IV.)

6. Invalid Obversion.—The most common error in obversion is the attempt to compensate for the change in the quality of the proposition by negating the *subject* term. (See Chap. IV.) This fallacy is closely related to that of denying the antecedent, just as invalid conversion is closely related to the affirmation of the consequent.

7. Four Terms.—This is a fallacy of the categorical syllogism. (See Chap. V.)

8. Ambiguous Middle.—A disguised form of the fallacy of four terms. There are, indeed, in this case, but three terms, if we have regard only to the words employed; but in passing from one premise to the other there is a shift in the meaning of the middle term.

9. Equivocation.—Not only the middle term of the categorical syllogism, but any term of any type of argument is in danger of being employed in more than one sense. Any error of this sort is called *ambiguity* or *equivocation.* (See Chap. VI.)

10. Two Negative Premises.

11. An Affirmative Conclusion from a Negative Premise.

12. A Negative Conclusion from Two Affirmative Premises.

13. Two Particular Premises.—Nos. 10-13 are fallacies of the categorical syllogism. (See Chap. V.)

14. Undistributed Middle.—This is a special case of the fallacy of ambiguous middle. If in each of the premises

only a part of the class denoted by the middle term is referred to, it is always possible that the part intended in the one premise is not the same as the part intended in the other. In the traditional logic, a term is said to be "distributed," in a given proposition, when the whole class denoted by it is referred to. Otherwise it is said to be "undistributed." Hence the name "undistributed middle" for the error of employing a syllogism in which the middle term is not distributed in at least one of the premises. An obvious example of this fallacy is the following: "All squares are polygons, and all triangles are polygons; therefore all triangles are squares." Manifestly the part of the class *polygon* which is referred to in the first premise is not the part referred to in the second.

15. **Illicit Major.**—The illicit or unlawful use of the major term. It consists in distributing the major term in the conclusion, when it was not distributed, i.e., known to be used in its complete extension, in the premise.

16. **Illicit Minor.**—The corresponding error in the use of the minor term. It is interesting to note that when the categorical syllogism is reduced to the hypothetical form, an undistributed middle frequently appears as the affirmation of the consequent, and an illicit major as the denial of the antecedent; while an illicit minor persists in the hypothetical syllogism as the attempt to make an assertion concerning the entire class denoted by the subject of the conclusion, in spite of the fact that only a part of this class is referred to in the premise.

17. **Accident.**—This is the error of supposing that what is true *generally* must also be true in peculiar or accidental circumstances; that what is true of a class must be true without any qualification of each and every member of the class. The classic illustration is the old puzzle of the "Epimenides." "Epimenides the Cretan says that all Cretans are liars. But

since Epimenides himself is a Cretan, it follows that he himself is a liar. Therefore we cannot believe him, and it is not true that all Cretans are liars." The puzzle may be solved in two ways, each of which illustrates the fallacy of accident. (1) Epimenides himself would probably have qualified his statement by some such principle as "present company excepted." In other words, while his statement has the form of a universal proposition, he probably did not mean it to be taken as strictly universal. (2) Even if Epimenides was a liar, his statement about the Cretans might nevertheless be true; for even a liar may now and then tell the truth. Mr. F. C. S. Schiller gives an illustration which is more modern.[1] "Suppose we assert that Smith loves good stories, does this imply that he loves good stories about himself?" Probably not; here again, while the proposition is in form universal, there is a tacit qualification. Casabianca remaining on the burning deck in obedience to his father's command exemplifies the fallacy of accident in affairs of action. His father never intended that he should "stay put" in that stupid fashion. In short, the fallacy of accident is the characteristic fallacy of stupid and literal-minded persons, who follow the letter rather than the spirit of that which is spoken or written.

18. The Converse Fallacy of Accident.—This is the error of supposing that what is true under peculiar or accidental circumstances must be true universally. For example, one might be tempted to argue that because a person when greatly fatigued ate some viand and was sickened by it, this particular article of food is indigestible: or because it is right in a time of extreme scarcity to help oneself to food belonging to another, stealing is not morally wrong.

19. Division.—This may be considered as a special form of the fallacy of accident. The error consists in supposing

[1] *Formal Logic*, p. 200.

that what is true of a whole must also be true of the parts. For example, it does not follow that because the Supreme Court is incapable of error in the interpretation of the constitution that therefore each justice of the court is incapable of error; it does not follow that because we must postulate the general trustworthiness of sense-perception, we must therefore assume that every report of our senses is veridical.

20. **Composition.**—This is the reverse of the fallacy of division. It would be exemplified if one should argue from the fallibility of each individual member of the Supreme Court to the fallibility of the Court as a whole; or from the possibility that any particular report of our senses is erroneous to the possibility that sense-perception is in general untrustworthy. Another example is the following: If the employees of the railroads could have their wages doubled, they would be better off financially; if the farmers could double the prices of their products, they would be better off financially; if professional men could double their salaries, they would be better off; etc., etc. Therefore, if *all* wages, prices, rates, salaries, etc., were doubled, everybody would be better off financially. Obviously, one increase would tend to offset the others. And it is frequently the case that what is true of individuals, considered separately, is not true of the aggregate of individuals, considered as a whole.

21. **The Misuse of Relative Terms.**—The error consists in forgetting that the meaning of terms such as rich, good, and the like, depends upon a standard of comparison. Suppose you are asked whether a man who is worth $100,000 is rich or poor; whether a mile is a long or a short distance. In either case your answer would depend upon the amount of money or the magnitude of the distance with which you compare the given amount or distance. What the other term of the comparison is to be, will be determined in each

case by the context. Thus if we compare your man of $100,000 with the Rockefellers, Rothschilds, or Morgans, he is poor indeed; while if you compare him with most of us, he is rich. In the same way, a mile is a considerable distance, if you are thinking of walks on the campus or in a small town, but short if you are thinking of travel on the railroad. No error is likely to arise in the use of relative terms as long as we remember that they are relative. But if we use these terms as if they had fixed meanings, like those of moon or chair, we are likely to fall into futile disputations. For example, it is futile to inquire whether a certain office building is high or low, big or little, until we have determined whether it is to be compared with the store and post office of a country town, or with the skyscrapers of the metropolis.

22. The Misuse of Abstract Terms.—This error arises when we treat a mere abstraction, like *wealth or goodness,* as if it were a *thing* or substance objectively existing. Thus the Platonist says that a white surface is white "because it participates in whiteness," as if the *whiteness* were an entity which could have objective existence apart from objects which are white. Under this head belong explanations which are merely "verbal," as if a boy, when asked to explain the manufacture of matches, should say that they "were made by machinery." Thus Molière's physician says that opium causes sleep, because it has "a soporific quality"; and many people would say that the apple falls to the ground "because of the law of gravitation." The adjective *soporific,* however, merely means "sleep-producing," and *gravitation,* instead of being the cause of the object's fall, is but a *name* for the observed fact that objects do fall when they are unsupported.

23. Hypostatization.—Another name for the fallacy of abstract terms. Derived from the Greek *hypostasis,* which

means *substance,* it means literally the mistake of treating a quality or attribute as if it were a substance.

24. Non Sequitur.—This phrase means literally, "it does not follow." It might accordingly be applied to any argument the conclusion of which is unsupported by the alleged premises. It is a convenient phrase to employ when one wishes to call attention to an obvious slip in the reasoning of an opponent, without saying precisely in what the error consists.

25. Petitio Principii.—This means "begging the question," or "arguing in a circle." The conclusion is smuggled in as a premise. The longer and more involved the argument, the more likely this is to occur.

26. Ignoratio Elenchi.—The ignoring of the precise point which is under discussion. Evading the issue. Setting out to prove one proposition, and instead proving another, which may, perhaps, be verbally similar to it.

27. The Argumentum ad Hominem.—An argument which is directed *to the man.* This is a very common form of the *ignoratio elenchi.* "When you have no case," the apocryphal barrister is said to have remarked to his young associate, "abuse the plaintiff's attorney." In strict logic the character of a man has no relevancy to a discussion of the truth of his opinions. Most people, however, are not so severely logical. And in political, theological, and even scientific discussion, it is almost impossible for most people to discuss the truth or falsity of propositions without taking into account the character of the men by whom these propositions are supported or opposed.

The phrase *argumentum ad hominem* is also employed for a mode of argument which approaches more closely to the *petitio principii.* A conclusion is recommended by an appeal to beliefs or prejudices entertained by the one who is to be convinced, but not universally received as true or

justified. In this case *the man* toward whom the argument is directed is the one who is to be influenced by the argument rather than the one whose opinion is controverted.

28. The Argumentum ad Populum.—An argument addressed to the passions and preconceptions of the crowd. Another form of the *ignoratio elenchi.*

29. The Argumentum ad Verecundian.—In this, which is also a form of the *ignoratio elenchi,* an appeal is made to the reverence which people have for a great name or a long-established institution. Instead of asking for the evidence, if any there be, by which the doctrine in question is supported, inquiry is made concerning the person or the institution by whom the doctrine has been inculcated. In short, there is here an appeal to authority instead of to reason.

30. The Argumentum ad Ignorantiam.—The fallacy consists in maintaining that something which cannot be *dis*-proved must therefore be true. Belief, however, cannot properly be based upon ignorance, but should have a foundation of positive evidence. (See Chap. XIV.)

31. The Argumentum ad Baculum.—This, the most flagrant of all the forms of irrelevancy, is the appeal to force. It is the argument of the bully and the persecutor. Opposed to all these forms of the *ignoratio elenchi,* is the *argumentum ad rem;* i.e., an argument addressed to the precise question at issue.

32. Many Questions, or Complex Question.—The stock example of this fallacy is the question, "Have you left off beating your wife's mother?" Clearly either "Yes" or "No" would be misleading. Not all questions can be answered by *yes* or *no.* Frequently a question contains an *assumption.* And before trying to explain an alleged fact, it is well to inquire whether the fact is really a fact. Thus if one were asked to explain why the earth is nearer the sun in June

than in December, the best answer would be that it isn't.

33. **Neglected Aspect.**—This is the fallacy that is likely to arise in the use of hypotheses. A hypothesis may satisfactorily account for all the facts which have been considered, but there is always danger lest some relevant fact has been overlooked.

34. **Post Hoc Ergo Propter Hoc.**—"After this, on account of this." We have no right to suppose that, just because one event takes place in close temporal connection with another, there is therefore a causal relation between them. A man takes a patent medicine and gets well; therefore the medicine has cured him. An accident happens on Friday; therefore Friday is an unlucky day. A college student "goes to the bad"; therefore colleges are nurseries of vice. In this way it is probable that many popular superstitions and unfair judgments concerning men and things have arisen.

35. **Hasty Generalization.**—Some of the examples of the preceding fallacy may also be taken as examples of hasty generalization. "One swallow does not make a summer," is the wise old adage which warns us against an over-readiness to generalize. In induction by "simple enumeration" one is especially likely to fall into this error. The canons of induction have been devised to guard us against it.

36. **Plurality of Points of Agreement.**

37. **Plurality of Points of Difference.**

38. **Accidental Concomitance.**

39. **Plurality of Causes.**—(For this and the three next preceding, see Chap. IX.)

40. **False Analogy.**—An inference based upon superficial resemblances. (See Chap. XI.)

41. **Idols of the Cave.**—This is perhaps, if we speak strictly, not a fallacy. It is one of the four classes of errors,

or sources of error, enumerated by Sir Francis Bacon. The "idols of the cave," as described by Bacon, are the errors which arise on account of personal idiosyncrasy. They are manifestations of individuality. The true investigator tries so far as possible to make allowance for his own "personal equation," and to find results which will be true for all observers.

42. **Idols of the Tribe.**—These, in Bacon's phraseology, are the errors which are common to men as men. We tend to take an "anthropocentric" view of our world. The true investigator seeks, so far as possible, to rise above the merely human point of view.

43. **Idols of the Market Place.**—These, according to Bacon are the errors which are due to the ambiguous use of language. "Words are wise men's counters, but a fool's money."

44. **Idols of the Theater.**—This name was employed by the same philosopher to characterize the errors caused by the spirit of party or faction. Even scientists and philosophers, consecrated as they ought to be to the pursuit of truth, are prone to divide into "schools" and "tendencies."

The Classification of Fallacies.—No satisfactory classification of fallacies has been devised, and none appears to be possible. For in many cases the error may be diagnosed in more than one way, depending upon the point of view from which the argument is regarded. Furthermore usage varies greatly in this matter; the names by which the various fallacies are known are used more widely by some writers than by others. Thus, as we have seen, the various fallacies frequently overlap. Nevertheless, if we remember that this chapter is a study of words and phrases rather than of things, it will be worth while to note certain divisions of fallacious reasoning which have been proposed.

Sophism and Fallacy.—A sophism may be defined as a

fallacy which is intentional. The speaker or writer who is accused of *sophistry* is alleged to employ an argument which he knows to be fallacious in the hope that the fallacy will not be discovered by his auditors or readers. The greater one's desire to edify or to persuade to action, the greater the temptation to resort to sophistical arguments. This is the besetting sin of the demagog and the moralist. The *argumentum ad hominem* and the *argumentum ad populum* are likely to be both sophisms and fallacies; although here, as in so many places, the line between sincerity and insincerity is very hard to draw.

Deductive and Inductive Fallacies.—It is also customary to make a distinction between the errors which occur in deductive thinking, and those which occur in induction. Thus, in the above list, Nos. 1-32 might be classified as deductive fallacies, and Nos. 33-44 as inductive. Inasmuch, however, as there is no hard and fast distinction between these general types of reasoning, it follows that it is impossible to make a sharp distinction between the deductive and the inductive fallacies.

Formal and Material Fallacies.—If an argument is formally valid, but one of the premises is untrue, the argument is said to be *materially* fallacious. In other words, a material fallacy is an error in the subject-matter; while a formal fallacy is an error in the structure of the argument of such a sort that, even if both the premises were true, the conclusion might nevertheless be false. For example, the *denial of the antecedent,* the *affirmation of the consequent, two negative premises,* and *two particular premises* are formal fallacies; while *incomplete enumeration, equivocation, accident, composition,* and *division* are material fallacies.

Whether *imperfect disjunction* is to be classified as a formal or a material fallacy depends upon our definition of

the *alternative proposition.* If it is understood that the alternatives are mutually exclusive as well as exhaustive, in other words that all alternative propositions are to be considered disjunctive, then a proposition in which both alternatives are true is an untrue proposition, and an argument based upon it is *materially* invalid. If, on the other hand, it is not our understanding that all alternative propositions are disjunctive, then the fallacy of imperfect disjunction is *formal* rather than material.

Useful as the distinction is, the line between formal and material fallacies breaks down in several places. Thus there is no sharp line between the formal fallacy of *four terms* and the material fallacy of *equivocation.* Again, whether we should include errors in obversion and conversion under formal fallacies depends upon our doctrine of "immediate inference." If we treat "immediate inference" as *inference* in the true sense of the term, these errors are formal fallacies. If, however, as seems to be a sounder view, we consider these operations as merely ways of *interpreting* a given proposition, then errors in obversion and conversion must fall under the head of material fallacies, or be set apart in a class by themselves.

Exercises.—1. The seamstress did not understand her trade, because she sewed the patch on the wrong side.

2. Every day has twenty-four hours; yet the days are longer in summer than in winter.

3. X is either A or B; it is A; therefore it is not B.

4. If X is A it is not B; it is not B; therefore it is A.

5. If M is N, P is Q; but M is not N; therefore P is not Q.

6. Some A is B; some M is A; therefore some M is B.

7. All A is B; all M is B; therefore some M is A.

8. Which of the following terms are relative? Point out the different standards of reference which may be employed.

(a) The horse ran rapidly down the smooth road until he got tired.

(b) Intelligent men would not expect untrained employees to do difficult work.

(c) The workmen protested against the long hours and the monotony of their labor.

9. Explain the confusion which results from overlooking the relativity of terms in the following:

(a) As a thing is generally sold for more than it is worth or for less, one of the parties to an exchange commonly is a loser by the transaction. (Hibben.)

(b) Everyone knows that a pound of gold is worth more than a pound of bread.

(c) He who is most hungry eats most; he who eats least is most hungry: therefore he who eats least eats most. (Aldrich.)

10. Which of the following terms are abstract and which are concrete: Desk? Angle? Angularity? Squareness? Square? Healthy? Health? Poor? Poverty? Whiteness? White? Benevolence? Obstinacy? Happy? Pretty? Beauty?

11. Write the abstract terms corresponding to each of these: old, young, ugly, costly, gentle, human, long, wide, valid, monotonous.

12. Write a concrete word or phrase corresponding to each of these abstract terms: antiquity, solemnity, sanctity, consciousness, charity.

13. Explain the fallacy in each of the following arguments:

(a) God did not make the world, because it was produced by evolution.

(b) Drugs are valueless in the treatment of disease, because we must at any rate depend on Nature to effect a cure.

14. A man tries to shoot a squirrel that is clinging to a tree on the side opposite to the man. The man walks completely round the tree, but the squirrel always keeps the trunk of the tree between itself and the man. Did the man go round the squirrel?

15. Which is the mother of the chick,—the hen that hatched it, the hen that is caring for it, or the hen that laid the egg from which it came (assuming that the three hens are different.)

16. Man is a rational being; therefore a lover always acts rationally.

17. Eggs and milk are wholesome foods; therefore they are good for fever patients.

18. Wine and brandy are sometimes beneficial in sickness; therefore they should be used moderately by everyone.

19. This stove saves half the ordinary amount of fuel; therefore two such stoves would save it all. (Bode.)

20. Freedom of speech is one of our most sacred privileges; therefore debate in the United States Senate should be unrestricted.

21. In going round the world westward, we keep gaining time, and the whole trip would gain us a full day; therefore if we could make the complete journey in twenty-four hours it would really take us no time at all.

22. Wine is a stimulant; therefore in every case where a stimulant is harmful, wine is harmful. (Sellars.)

23. Improbable events happen almost every day; events which happen almost every day are probable events; therefore improbable events are probable events.

24. These men are traitors, because they are opposed to the war; for opposition to the war is opposition to the government, and opposition to the government is traitorous.

25. This man always votes the democratic ticket: therefore he is democratic in his relation to his fellowmen.

26. Big business is in favor of the World Court and the League of Nations. Therefore I am opposed to them.

27. Washington cautioned his contemporaries against entangling alliances; therefore, if he could advise us to-day, he would say, Keep out of the League of Nations.

28. The people are the legitimate source of power; therefore universal suffrage is among the natural rights of man.

29. We should love our neighbors more than ourselves; yet everyone loves his immediate relatives more than strangers.

30. A gentleman told me that he had a conclusive argument for opening the Harvard Medical School to women. It was this: 'Are not women human?'—which major premise, of course, had to be granted. 'Then are they not entitled to all the rights of humanity?' My friend said that he had never met any one who could successfully meet this reasoning. (James, *Psychology;* quoted by Bode.)

31. He who cannot possibly act otherwise than he does has

neither merit nor demerit in his action; a liberal and benevolent man cannot possibly act otherwise than he does in relieving the poor; therefore such a man has neither merit nor demerit in his action.

32. No reason, however, can be given why the general happiness is desirable, except that each person, so far as he believes it to be attainable, desires his own happiness. This, however, being a fact, we have not only all the proof which the case admits of, but all which it is possible to require, that happiness is a good, that each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons. (Mill, *Utilitarianism.*)

33. I am under an obligation to do it; but he who is obliged has no power of resistance; consequently I have no choice about the matter.

34. I may fairly expect that one who has received kindness from me should protect me in distress; yet I have reason to expect that he will not. (Whately.)

35. To be up after midnight, and to go to bed then, is early; so that to go to bed after midnight is to go to bed betimes. (Shakespeare, quoted by Bode.)

36. No soldiers should be brought into the field who are not well qualified to perform their part; none but veterans are well qualified to perform their part; therefore none but veterans should be brought into the field. (Whately.)

37. The sun moves to the south because of the cold which drives it into the warm part of the heavens over Libya. (Herodotus.)

38. "Do unto others as you would have others do unto you." If I were unable to answer the questions in an examination, I should want my neighbor to give me assistance; therefore it is my duty to help this man who is having trouble. (Bode.)

39. "What is worth doing at all is worth doing well." Show how this statement must be limited in its application. (Bode.)

40. The whole is greater than any of its parts; we are capable of wisdom and we are part of the world; therefore the world is capable of wisdom.

41. Actions that benefit mankind are virtuous; therefore it is a virtuous act to till the ground. (Jevons.)

42. The end of a thing is its perfection; death is the end of life; therefore death is the perfection of life.

43. It is the business of the State to enforce all rights; a judicious charity is right; therefore it is the business of the state to enforce a judicious charity. (Joseph.)

44. Do you know Coriscus? Yes. Do you know the man approaching you with his face muffled? No. But he is Coriscus, and you said you knew him. (Aristotle; quoted by Joseph.)

45. Freedom is good; therefore every nation should have the right of self-determination.

46. I have the right to publish my opinions concerning the present administration; what is right for me to do, I ought to do; therefore I ought to publish my opinions concerning the present administration. (Hibben.)

47. Whatever menaces the public interests should be prevented by law. The Great Northern Securities Merger menaces the public interests. Therefore it should be prevented by law. (*Ibid.*)

48. The board of directors of the XYZ Company is very conservative. Mr. S is a member of the Board. Therefore he is very conservative.

49. Strychnine is a deadly poison, and therefore it can never be used as a medicine. (Hibben.)

50. Henry Ford, Thomas A. Edison, and Charles M. Schwab are successful men. They never went to college. Therefore a college education is not conducive to success in life.

51. The nature of heavy things is to tend to fall toward the center of the universe, and of light things to fly from it; therefore the center of the earth is the center of the universe. (Aristotle.)

52. The bill before the House is well calculated to elevate the character of education in this country, for the general standard of instruction in all our schools will be raised by it. (Hibben.)

53. That which is rare is dear; cheap horses in Paris are rare; therefore cheap horses in Paris are dear. (Toohey.)

54. A man has a right to walk in the city; a murderer is a man; therefore a murderer has a right to walk in the city. (*Ibid.*)

55. All good fathers provide food and clothing for their children;
Mr. B. provides food and clothing for his children;
Therefore Mr. B. is a good father. (Schuyler.)

56. All moral beings are accountable; no brute is a moral being; Therefore no brute is accountable. (*Ibid.*)

57. All cold can be expelled by heat; John's illness is a cold; Therefore it can be expelled by heat. (Turner.)

58. What you bought yesterday, you ate to-day; you bought raw meat yesterday; therefore you ate raw meat to-day.

59. It is unlawful to take another man's life. But the soldier in battle takes another man's life. Therefore what the soldier does is unlawful. (Turner.)

60. The volume of a body diminishes when it is cooled, because the molecules then become closer. (*Ibid.*)

61. All great musicians practice every day; I practice every day; therefore I am a great musician.

62. Meanwhile, in the face of the appeal to the church people and what might be called the professional advocates of the court and the League of Nations, the public is gradually becoming aware that some of the most noted supporters of court and league are the worst standpatters and reactionaries we have. Some of the very men who are most conspicuous in their efforts to jail all who differ with their views, and who are opposed to all thoroughgoing domestic reforms, demand that we save Europe by means of the court. (*The Nation,* Vol. CXVI, p. 733.)

63. Say not thou, What is the cause that the former days were better than these? For thou dost not inquire wisely concerning this. (Ecclesiastes 7: 10.)

64. "These peas made me feel ill. I can tell when I feel ill, can't I? Why do you question it then?" (Snow, *Problems in Psychology,* p. 34.)

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CHAPTER XIV

THE PRESUPPOSITIONS OF SCIENCE

Questions.—Before reading the subsequent discussion, the student is advised to formulate answers to these questions:

1. Can you recall any instances in which your senses deceived you?
2. How did you discover your mistake?
3. Did you ever feel perfectly sure that your recollection of a certain event was correct, only to discover afterwards that you were in error?
4. How were you convinced of your error? By comparing your recollection with that of some reliable witness or witnesses? By comparing it with written records?
5. Did you ever "prove" a point to your complete satisfaction, and discover later that your reasoning had not been cogent?
6. How did you detect this lapse of reason?
7. How do you know that a dog which you now see for the first time eats meat?
8. How do you know that fire will melt ice to-morrow?
9. Does everything which takes place have a cause?
10. What meaning is suggested to your mind by the adjective *impossible?*
11. May a proposition which is false from the point of view of human logic be true from some higher point of view?
12. Does one have a right to believe a proposition which can not be proved?
13. Is one under obligation to believe every proposition which can not be disproved?

Skepticism.—A skeptic is one who doubts. The term is derived from a Greek word which means *to look carefully.*

In ancient times there were several skeptical schools. Of these the most important was the Pyrrhonist, so called from Pyrrho of Elis, a Greek philosopher who lived about 300 B.C. He is said to have carried his skepticism so far that he even "doubted whether he doubted." Carneades, a leader of the Middle Academy, who lived about a century later than Pyrrho, was less extreme. His position seems to have been that absolute certainty is unattainable, and that our lives must therefore be based upon *probability*. This position is not far removed from that of many contemporary logicians, and is indeed substantially the position taken by this book. A skeptic is not necessarily a bad man. And skepticism should not be condemned without examination. The skeptic despairs of the possibility of knowledge, because he distrusts human faculties and does not see by what right we pass from necessities of our thinking to necessities of being. We shall find that his position is a mixture of truth and error.

A Three-Fold Doubt.—The questions at the beginning of this chapter suggest a number of perplexities. Nos. 1-6 relate to the reliability of sense-perception, memory, and reason; Nos. 7-9 to what is commonly known as the "uniformity of nature." While the remaining questions insinuate a doubt concerning the validity of logic as an index to the nature of reality. In what follows we shall not try to refute skepticism. Instead, we shall concede its partial truth, and point out certain "postulates" which science is compelled to make, but cannot prove.

The General Trustworthiness of Sense-Perception.—The skeptic is impressed by the illusions of sense-perception. The rails of the railroad track appear to converge in the distance; yet if we thought that they really converged, we would never get on the cars. We say the appearance is only an illusion, and we are not deceived by it. There are cases, however, in which our senses really lead us into error.

And the skeptic asks whether, since the senses have been shown to be deceivers in some cases, we can be certain that they are reporting truthfully in any case. In short, he insists that we can never be sure that sense-perception is genuine. Is there any way in which we can turn the edge of the skeptic's objection?

Our solution of the difficulty may be put into the form of a paradox: *in order to convict the senses of error, we must assume them to be veracious.* How do we know that the rails do not really converge? Because, if we walk along the track to the place where they seemed to come together, they will be *seen* to be just as far apart there as anywhere else. Or, again, to take another hackneyed example of illusion in sense-perception, how do we know that a stick is not really bent at the surface of the water, as it appears to be? Only because, if we run our hand along it, the stick *feels* straight: and if we pull it out of the water (or push it completely under the water), it *looks* straight. In short, consider any case which you choose of alleged error in sense-perception, and you will find that the fact of error can be established in no other way than by comparison with other results of sense-perception; and these latter must therefore be assumed to be veridical.

This sort of reasoning does not, of course, prove that sense-perception is *always* trustworthy. Neither does it prove with complete cogency that *any* of our perceptions are veridical. All that it proves is that our right to *postulate* the general trustworthiness of our senses cannot be successfully assailed; since it is only by means of the senses that such errors as arise in sense-perception can be detected. In every given case, in other words, there is a *presumption* in favor of our sense-data; and he who would attack any of the results of sense-perception, must accept the burden of proof.

The General Trustworthiness of Memory.—Memory, too, is notoriously unreliable. Again and again we find that our recollections have been false or inaccurate. The skeptic suggests, accordingly, that no proposition which depends upon our memory or upon testimony is worthy of credence. But stop a minute! How do you know that your memory has ever deceived you, or that the testimony of others has been erroneous or mendacious? Only by comparison with other recollections. Only by memory can memory be proved false. For example, let us say that the evidence given by a certain witness is false. This can only mean that it disagrees with the account of the same event which is given by some other witness or witnesses. Or suppose that my own memory has misled me in regard to some past event. This means that my recollection of the affair is inconsistent with that of other people, or with other facts which I myself remember, or, perhaps, with written records.

This last case—the correction of memory by reference to written records—may seem at first to introduce a different principle. But, in reality it does not; here too the correction of memory presupposes the veracity of memory. My present recollection is shown to be erroneous, let us say, by reference to my diary. But that the diary is really mine, that I actually wrote what is found therein, that the characters employed in the writing were understood by me at the time of writing in the same way in which I now interpret them,—all these and many other necessary links in my present chain of reasoning must be accepted on the authority of memory.

The General Trustworthiness of Reason.—We have seen that errors in sense-perception cannot be detected without the aid of sense-perception, and that errors in remembering cannot be detected without the aid of memory. In the case of reason, too, the test may be homoeopathic. The

modern man, it is true, verifies the results of abstract thinking by reference to facts of experience. But here too the error may be detected by the same "faculty" which has produced it. The faults of reason may be self-cured. Thus in geometry we pronounce a demonstration invalid when some step in the reasoning contravenes other processes of reasoning. By the use of reason we prove that reason has erred.

The Impossibility of Theoretical Skepticism.—So long as the skeptic contents himself with a passive doubt his position is invincible. When, however, the skeptic attempts to show by argument that doubt is the only reasonable attitude, that assured knowledge is impossible, that skepticism is the only true philosophy, he soon becomes entangled in the mazes of self-contradiction. For he that would play a game must abide by the rules of the game. And, if a man wishes to think, and especially to argue, he must accept the presuppositions which are required to make argument, or any kind of thinking, possible. In brief, he must "postulate," that is to say, assume, take for granted, the general trustworthiness of our cognitive processes. Not that sense-perception, memory, and reason are infallible. Errors frequently occur. We postulate, not the *universal,* but merely the *general* reliability of our intellectual "faculties." The burden of proof rests upon him who holds that error has been committed. Every cognitive act is assumed to be veridical until it is proved erroneous.

The Postulate of Kinds.—In addition to the postulate of the general trustworthiness of our intellectual operations, there are at least three other postulates which are required by the science of nature. These are the *postulate of kinds,* the *postulate of permanence,* and the *postulate of determinism.* The first of these has already been touched upon.[1]

[1] See Chap. XI.

We have seen that arguments based upon analogy, and consequently all forms of inference, depend upon the postulate that resemblance is an index of further resemblance. Now, if qualities were distributed among objects merely at random, this postulate would be contrary to fact. In that case, likeness in certain respects, that is to say, the possession by two or more things of the same qualities, would not be a sign of likeness in other respects. Analogy and classification depend upon the postulate that resemblances go in flocks. This is called the "postulate of kinds" or natural classes, because if it were contrary to fact, classification would be completely arbitrary, and inferences based upon common membership in a class would be impossible.

The Postulate of Permanence.—In addition to the "postulate of kinds," the application of thought to the real world requires the *postulate of permanence*. Our thinking about the world depends upon the belief that what has been will be. We take it for granted that the laws of nature which are true to-day will be true to-morrow. But what assurance do we have, or can we have, that even the law of gravitation will hold next week, or next minute? What assurance do we have that particles of matter which are now attracting each other with a force proportioned directly to the product of their masses and inversely to the distance between their centers, might not begin to-night at 12 o'clock to repel each other in the same or some other ratio? It is manifest that we have no rational assurance whatsoever that such a reversal will not take place. Or, if some substance, such as iron, has been found to have certain properties—valence, melting-point, specific heat, etc. —, how do we know that these will not change over night? Clearly our feeling of assurance in these matters has no demonstrable basis. It is of the same sort as our confidence in the character of a friend. In general, then, we can but

postulate, or take for granted, that the past is an index of the future.

The Postulate of Determinism.—Closely related to the "postulate of kinds" and the "postulate of permanence" is the *postulate of determinism.* This is the assurance that *changes* are so related that one of them may be taken as a sign or cause of another. Another way of putting this is to say that antecedent differences are indices of subsequent differences, and the latter of the former. The subsequent change is commonly called the "effect" and the antecedent change the "cause," and it is said that "every effect has a cause." Thus stated the principle is tautologous. Certainly every *effect* has a cause; for to have a cause is included within the very definition of "effect." On the other hand, the statement that every *change* or every *event* has a cause, is far from self-evident. There is, indeed, no way in which it can be rationally established. It is conceivable that things might originate *de novo.* Events might be uncaused. At any rate, there is no *a priori* reason for refusing to admit the possibility that *some* events are uncaused.

The doctrine that every event has a cause is known as "determinism." The opposing doctrine is called "indeterminism." Whether thorough-going determinism is the true view of the world, or whether partial indeterminism is the true account, we are not now concerned to inquire. Our present point is that *science* is possible *only so far as* determinism is the true hypothesis. There may be undetermined events; but if they occur, they fall beyond the scope of science. The work of science is to predict and to explain, and undetermined events are in the very nature of the case unpredictable and inexplicable. Even ethics requires the postulate of determinism to give meaning to the concept of character. Moral training depends upon the assumption that the past in some way determines the future. Penology pre-

supposes that behavior is determined by the penalty inflicted. Modern psychology is founded upon the hypothesis of response to situation. And, in general, all the sciences presuppose the cause-effect relation. The very concept of *nature* as a realm of law implies determinism.

The Uniformity of Nature.—The postulates of kinds, of permanence, and of determinism together constitute the postulate of the "uniformity of nature." Unless nature is assumed to be uniform, no science of nature is possible. It is not necessary, however, that nature should be *rigidly* uniform. Whether or not it is, is a subject for further inquiry. In the degree that nature is uniform, in that same degree science is possible. In the degree that nature is not uniform, in that same degree science is impossible. The logic of science requires that classification should be regarded as possible for the most part; and that if fundamental changes are indeed taking place in the world, these changes are not so rapid nor so abrupt as to nullify the laws which have been discovered. The logic of science must assume that events are generally, if not universally, determined. In other words, just as our belief in the general trustworthiness of our intellectual processes is not incompatible with the recognition of occasional errors in sense-perception, memory, and reason, so our belief in the uniformity of nature is not incompatible with the recognition of the possibility, or even the probability, that there are *uncaused events,* and that some of these uncaused events enter as causes into the web of nature.

On the other hand, the doctrine that there are such "new beginnings" is a hypothesis which has no right to preferential treatment. Whether it or the rival hypothesis of thoroughgoing determinism is to be chosen, must depend upon their relative success in explaining the facts. In order, however, that physical science may be possible, events must

be assumed to be *sufficiently* correlated with those which went before to enable us to formulate laws of nature.

The Relation of Thought and Being.—If now we assume the trustworthiness of our cognitive processes, and take for granted the uniformity of nature, we may still inquire whether the universe is fundamentally intelligible. What is the relation of our rational processes to reality? Have we a right to assume that the necessities of our thought are also *necessities of being?* We are, of course, unable *to prove* that this is the case. Nevertheless we must postulate this correspondence; for otherwise science would have no relevancy to the real world. There are two methods by which we seek to learn the nature of the real world. One of these is positive; the other is negative. The former is the method of hypothesis, which we have already discussed, and to which we shall return at the end of this chapter. The latter, the negative method, may be called the method of conceptual analysis. The former of these methods depends upon the law of parsimony, and postulates that reality is *fundamentally simple;* the latter depends upon the law of contradiction, and postulates that reality is *free from contradiction.*

The Method of Conceptual Analysis.—If someone should tell us that on a certain island in mid-ocean there is a circular parallelogram, none of us would go to see whether he was telling the truth. For we are sure that no such geometrical figure will ever be found, either on his island or anywhere else. The notion of a "circular parallelogram," we should tell him, belongs to the same category as that of two neighboring mountains without an intervening valley; or that of an irresistible force which meets an immovable body; or that of a person who after living forty years is only twenty years old; or that of an arithmetic in which 2 plus 2 make 5. All of these are self-contradictory

concepts, and therefore they are unreal. The elements which enter into such concepts are mutually *inconsistent, incompatible,* or *logically repugnant.* (All of these adjectives are employed to indicate the relation of elements which are mutually contradictory.) A notion or concept which contains such incompatible elements is said to be *absurd, logically impossible,* or *inconceivable.*

Now the fact that a concept is *conceivable* is not a sufficient proof that it exists. It may be conceivable and yet be non-existent. There is nothing absurd, for example, about the concept of a mountain of pure gold. So far as logic goes, the idea is entirely possible. Yet no such mountain exists. In like manner, fairies, kobolds, mermaids, etc., are logically conceivable, but are not thereby proved to be existent.

But, while we cannot pass from the conceivability of a concept to its existence, we usually assume that we may pass from *in*conceivability to *non*-existence. If a notion can be shown to be logically absurd, we inevitably go further, and conclude that it is also contrary to fact. This is what is meant by saying that the method of conceptual analysis is negative. It has the power of veto, but not that of positive enactment.

The relation between conceivability and existence may be expressed in the form of a hypothetical proposition.

If x exists, then x is conceivable.
Existence < Conceivability.

We cannot argue from conceivability to existence; for that would be the fallacy of "affirming the consequent." But we can argue from *in*-conceivability to *non*-existence, for that is a case of *modus tollens.*

Inconceivability as an Index of Non-Existence.—Before examining further this doctrine that inconceivability is

a negative test of reality, we must guard against three insidious confusions. (1) We should not confuse the logically impossible or absurd with the merely *improbable*. It is highly improbable that I shall to-morrow receive a gift of a million dollars; but it is not logically impossible. (2) We should not confuse logical impossibility with *physical* impossibility. It is physically impossible that any living man should lift a weight of a ton; but there is no logical incompatibility between the idea "man" and the idea of ability to lift a ton. (3) We should not confuse logical impossibility or absurdity with our inability to *imagine*. Conceivability is not the same as *imaginability*. We cannot imagine a plane figure of ten thousand sides; yet such a figure is perfectly conceivable, for the idea contains no elements which are contradictory.

It is not, then, in the loose sense of the unimaginable, the physically impossible, or the improbable, that we employ the idea of the inconceivable as an index of non-existence; but in the strict sense of the logically inconsistent or absurd.

"Antinomies" of Reason.—Even in this strict sense, however, the criterion has been challenged. Philosophers have alleged the existence of what they call "antinomies," i.e., inevitable oppositions of reason to itself. Thus Kant maintains that it is possible to prove with equal cogency both that the world had a beginning and that it did not have a beginning; both that matter is composed of indivisible units and that it is indefinitely divisible, etc. If such antinomies really exist, what becomes of our inference from inconceivability to non-existence? Perhaps human reason is unable to comprehend the ultimate principles of reality!

Apparent vs. Real Inconceivability.—And, indeed, we must admit, as a theoretical possibility, that some propositions which appear contradictory to human reason might be conceivable from the point of view of a higher intelligence.

Nevertheless, while we may admit that an appearance of contradiction does not necessarily indicate contradiction as viewed by an all-knowing mind, we must insist that it is this self-same *apparent* inconceivability which is our only available sign of real inconceivability. Let us not forget that all philosophy must needs be written *from the human point of view.* And the tests proposed by human logic are the only tests we have. Accordingly, unless we are to be without defence against the seductions of error, we must *postulate* the trustworthiness of the principle of contradiction as a negative test of real knowledge. For, if we employ this test against views which we regard as erroneous, we cannot reasonably refuse to admit its justification when our own favorite doctrines are haled into court.

Reality and the Law of Parsimony.—The method of hypotheses we have called the "positive" method of gaining insight into reality. The hypothesis which the better agrees with, or "explains" the relevant facts is said to be true.[1] Whether to be true is merely to "work" satisfactorily, or whether its "working" is but an evidence of the truth of the hypothesis, is a question which we need not attempt to decide. If, however, we should adopt the latter point of view, a further problem would arise. What right do we have to assume that the simpler hypothesis is the true hypothesis? What is there about this attribute of simplicity which gives it this portentous significance? Perhaps the world is *complex,* and not simple! And why, indeed, should human convenience be the criterion of reality?

It must be admitted that to these questions there is no answer, except to say that, if we would reason about the real world at all, we must postulate the law of parsimony. We have seen that the method of conceptual analysis is negative. At the most it gives us nothing more than a world of

[1] See Chap. VIII.

ideals; for concepts may be self-consistent without being real. Only by the method of hypotheses can we extend our thinking to reality. Now if we are to use the method of hypotheses, we certainly cannot treat *the most complex* hypothesis as true. And no other satisfactory criterion is available. Consequently we must fall back upon the criterion of relative simplicity. We must choose our hypotheses so as to explain in the simplest way the great possible number of facts. The world-view which accounts for all the verifiable facts with the smallest number of assumptions must be regarded as the true theory of the universe.

The Argumentum ad Ignorantiam.—The refusal to accept the law of parsimony leads to the fallacy which is known as the "argumentum ad ignorantiam." It consists essentially in the demand for a demonstrative *dis*proof of a doctrine or opinion. Usually the doctrine or opinion is one which in one way or another has become emotionally precious. No satisfactory evidence is at hand in favor of the cherished belief; yet its protagonist refuses to give it up, and demands that others shall adopt it, for the reason that *it cannot be disproved.* Thus one who believes in the transmigration of souls, or the existence of many gods, or many another favorite doctrine, may argue that the doctrine should be accepted on the ground that there is no good reason *against* it. The following argument strikingly illustrates this fallacy:

"Beyond the orbit of the planet Neptune, the last of the planets, at a distance many millions of miles greater than that of Neptune from the sun, there is a silver teapot which revolves regularly about the sun just as the planets do, although with a much longer 'year.' To be sure, no one has ever seen the teapot. It is so small as to be invisible with the most powerful telescope. The effect which its attraction produces upon other bodies is so slight as to be incapable

of detection by any means at our disposal. Yet we defy anyone to prove that this teapot does not exist, and that it does not encircle the sun in the manner suggested. Our inability to perceive it, or to find any evidence for its existence, is due to the limitations of our human faculties of observation, even when assisted by the best helps at our disposal. A greater intelligence than ours is no doubt able to perceive it. Therefore, unless we wish to convict ourselves of arrogance and presumptuous unbelief, let us believe in the existence of the silver teapot."

The reasoning is obviously fallacious. The fallacy consists in the attempt to base a demonstration upon ignorance. We cannot prove the nonexistence of the teapot; but the principle of parsimony forbids us to accept the hypothesis of its existence, *unless there is some positive evidence for it,*— some evidence, moreover, which cannot be explained as well by some other hypothesis.

Summary.—In this chapter we may have strayed across the imaginary line which separates logic from epistemology.[1] We have discussed the relation of thought to the world of independent reality. We have seen that it is necessary for him who would think about the real world to postulate the general trustworthiness of sense-perception, memory, and reason. He must also accept the postulate of kinds, the postulate of permanence, and the postulate of determinism; and must take for granted that, if it is to be true, a description of reality must be consistent and relatively simple. In other words, he must postulate the fundamental intelligibility of the universe.

Exercises.—Compare the point of view expounded above with that of the following quotations:

1. The ephemeral nature of scientific theories takes by surprise the man of the world. Their brief period of prosperity

[1] The technical name for the theory of knowledge.

ended, he sees them abandoned one after another; he sees ruins piled upon ruins; he predicts that the theories in fashion to-day will in a short time succumb in their turn, and he concludes that they are absolutely in vain. This is what he calls the *bankruptcy of science.*

His scepticism is superficial; he does not take into account the object of scientific theories and the part they play, or he would understand that the ruins may be still good for something. No theory seemed established on firmer ground than Fresnel's which attributed light to the movements of the ether. Then if Maxwell's theory is to-day preferred, does that mean that Fresnel's work was in vain? No; for Fresnel's object was not to know whether there really is an ether, if it is or is not formed of atoms, if these atoms really move in this way or not: his object was to predict optical phenomena. This Fresnel's theory enables us to do as well to-day as it did before Maxwell's time.

. . . The kinetic theory of gases has given rise to many objections, to which it would be difficult to find an answer were it claimed that the theory is absolutely true. But all these objections do not alter the fact that it has been useful, particularly in revealing to us one true relation which would otherwise have remained profoundly hidden—the relation between gaseous and osmotic pressures. In this sense, then, it may be said to be true. (Poincaré, *Science and Hypothesis,* p. 161ff.)

2. Two opposite tendencies may be distinguished in the history of the development of physics. On the one hand, new relations are continually being discovered between objects which seemed destined to remain forever unconnected; scattered facts cease to be strangers to each other, and tend to be marshalled into an imposing synthesis. The march of science is towards unity and simplicity.

On the other hand, new phenomena are continually being revealed; it will be long before they can be assigned their place —sometimes it may happen that to find them a place a corner of the edifice must be demolished. In the same way, we are continually perceiving details ever more varied in the phenomena we know, where our crude senses used to be unable to detect any lack of unity. What we thought to be simple be-

comes complex, and the march of science seems to be towards diversity and complications.

Here, then are two opposing tendencies, each of which seems to triumph in turn. Which will win? If the first wins, science is possible; but nothing proves this *a priori,* and it may be that after unsuccessful efforts to bend Nature to our ideal of unity in spite of herself, we shall be submerged by the ever-rising flood of new riches and compelled to renounce all hope of classification—to abandon our ideal, and to reduce science to the mere recording of innumerable recipes. (*Ibid.*, p. 172f.)

3. Metaphysical hypotheses progress from the simple to the complex. Each new fact discovered adds its quota to the irreconcilable and conflicting properties of the ether and the atom, and these invisible links of the universal machine grow more and more bewildering and complicated, until the whole construction falls to pieces. Nor is this all; the man of science forgets that he is building toy houses, and ends by believing in their reality. Even if hypothesis does not carry him so far, it certainly has this effect on others who accept the dogmas of science without discrimination. It is no small danger thus to confuse reality and imagination; a science, which becomes so hypothetical or so specialized as to be unintelligible to the educated man, is apt to become as sterile as a religion which is the sole possession of a hierarchy.

This excessive use of hypothesis has developed a sort of scientific cult which somewhat resembles a religious dogma, in that adverse criticism of either arouses a feeling of personal irritation. The rancor of religious polemic is well known and is said to be due to the fact that the believer of a religion relies on revealed truth, to doubt which is sinful. This same occurs with the supporters of a scientific hypothesis, who declare their system to be founded on objective, experimental fact, and to be developed by logical methods, so that in doubting the hypothesis we are sinning against truth and reason,—the gospels of science. On the other hand, discussions in experimental science are noted for their calmness, for then we are criticizing, not personal opinions but objective facts, and we care comparatively little which way the matter ends. The theorist, on the contrary, forgets that while founded on experience, his hypothesis is developed in one way or another according to his

own personal opinion; for example, the same facts of light made Newton believe in corpuscles and Huygens in waves, and so the theorist injects into his discussions the bitterness of personal defeat or the exultation of personal victory. (More, L. T., *The Limitations of Science,* p. 34f.)

4. It appears that certain characteristics of man's procedure in aiming at the control of his experience stand out more clearly in his religious than in his scientific attitude. For example, the fact that faith everywhere precedes knowledge. No doubt it is true that science also ultimately rests on acts of faith, that the assumptions, which encourage us to control and understand the world must be risked before experience can confirm them. Thus no order of events, however regular, could ever prove the existence of "laws of nature" to an intelligence which held this sequence of events to be fortuitous. For such an intelligence would not connect them, and so would see no reason why the sequence should not stop, or go on, at any point and in any way.

The unity of Space and Time, the indestructibility of "Matter" and the conservation of Energy, nay, the very unity of the "universe" itself, are similarly postulates. Science, therefore, indubitably starts from postulates which are envisaged by Faith before they are proved by experience. It is mere verbalism (as well as sheer negation of Science) to claim to have removed the risk of postulates by calling them *a priori.* For even *a priori* necessities of thought do but conceal the risk, and cannot dispense with an act of Faith which believes in their continuance. But it is true that, until recently, Science was not so keenly conscious of its acts of Faith, and (when misguided by philosophers) even imagined that pure reason or unaided experience would establish its foundations. No religion has ever made such a pretence of dispensing with Faith, or of laying claim to a purely rational demonstration. (Schiller, F. C. S., *Riddles of the Sphinx,* third edition, p. 466.)

5. It is our duty to think of God as personal; and it is our duty to believe that he is infinite. It is true that we cannot reconcile these two representations with each other; as our conception of personality involves attributes apparently contradictory to the notion of infinity. But it does not follow that this contradiction exists anywhere but in our own minds: it

does not follow that it implies any impossibility in the absolute nature of God. The apparent contradiction, in this case, as in those previously noticed, is the necessary consequence of an attempt on the part of the human thinker to transcend the boundaries of his own consciousness. It proves that there are limits to man's power of thought; and it proves no more. (Mansel, *Limits of Religious Thought,* third edition, p. 106.)

6. This leads me to the question, whether belief in a mystery can be more than an assertion. I consider it can be an assent, and my reasons for saying so are as follows:—A mystery is a proposition conveying incompatible notions, or is a statement of the inconceivable. Now we can assent to propositions (and a mystery is a proposition), provided we can apprehend them; therefore we can assent to a mystery, for, unless we in some sense apprehend it, we should not recognize it to be a mystery, that is, a statement uniting incompatible notions. The same act, then, which enables us to discern that the words of the proposition express a mystery, capacitates us for assenting to it. . . . An alleged fact is not therefore impossible because it is inconceivable; for the incompatible notions, in which consists its inconceivableness, need not each of them really belong to it in that fulness which would involve their being incompatible with each other. (Newman, *Grammar of Assent,* p. 45 and p. 51.)

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CHAPTER XV

PROPOSED CRITERIA OF TRUTH

Theories of Truth.—Consider any proposition which you believe to be true, and ask yourself why you believe it. Is it true because you are able to prove it, either syllogistically, or in accordance with the canons of induction? Or, is it true because you have read it in a certain book or heard it from the lips of a certain teacher? Or, once more, is it true because you just know it is true,—because you feel absolutely certain about it, without any argument or discursive thought whatsoever, and without depending upon any external authority? Various theories have been held concerning the standard or standards to which we should appeal in deciding whether propositions are true or false. Among these theories are *rationalism, authoritarianism, mysticism, intuitionism,* and *pragmatism.*

According to rationalism, a proposition is true if it fulfils the requirements imposed by reason. Just what this means we shall presently inquire.

Authoritarianism, on the other hand, is the theory that truth is determined by some authority which is superior and external to the individual inquirer.

Mysticism, like rationalism, holds that the authority in matters of truth and falsity resides, not in any book or man or institution, but within the experience of the inquirer himself; mysticism, however, in opposition to rationalism, prefers that which is immediately apprehended to that which is obtained through the operation of "discursive" reason.

Intuitionism agrees with mysticism in emphasizing the importance of immediate apprehension; but thinks of these immediate apprehensions or "intuitively known truths," as belonging to the ordinary experience of all normal human beings, rather than to those moments of extraordinarily clear insight, which, according to the mystic, are sometimes granted to a few exceptional individuals.

Pragmatism is the theory that a proposition is true if it "works"; that the truth of a doctrine consists essentially in its utility.

Rationalism.—These preliminary statements are, of course, inadequate. We shall now discuss these theories at greater length. To take them in order, rationalism may be characterized as the point of view of logic. To allow our opinions to be determined by the authority of an institution, a book, or a teacher, is, from the point of view of strict logic, to be guilty of the *argumentum ad verecundiam* or the *argumentum ad hominem*. If we wish to know whether a doctrine is true, we ask for the evidence; to ask who has taught the doctrine is an evasion of the issue. No matter how venerable the teacher may be, it is both the right and the duty of the learner to weigh the evidence for himself, and to come to an independent decision. Truth is to be discovered by reason. This is rationalism as contrasted with authoritarianism.

The term rationalism, however, is also employed as the opposite of *sensationalism or empiricism*. The issue between rationalism and empiricism is, however, not the same as that between rationalism and authoritarianism. Empiricism is a theory about the *origin* of knowledge, and not concerning the *standard* by reference to which knowledge is to be judged. It is important that we should not confuse the two questions. A man may, indeed, be an empiricist in his answer to the one question, and a rationalist in his an-

swer to the other. And this, in fact, is the position of John Locke. As regards the problem of the origin of knowledge, he is an empiricist; for he denies the existence of "innate ideas," and stoutly insists that all knowledge comes from experience. As regards the problem of the *test* of knowledge, however, Locke is a rationalist; for he defines genuine knowledge as the "perception of the connection and agreement, or disagreement and repugnancy, of any of our ideas." [1] And not only Locke, but nearly all modern thinkers are rationalists in their doctrine of the test of truth. Indeed, rationalism in this sense is one of the outstanding differences between modern and mediæval thought. The mediæval thinker may be likened to a school boy "doing his sums" in arithmetic. He must get the answers in the back of the book. For the modern thinker, on the contrary, there are no answers in the back of the book.

Correspondence and Coherence.—When we inquire more precisely concerning the test of truth upon which the rationalist depends we meet two answers. One of these is that ideas or propositions are true if they are good copies of reality; the other that they are true if they can be included in an organized whole of knowledge. The former is known as the "correspondence theory"; the latter as the "coherence theory." For example, the proposition, "A is between B and C," is true, according to the correspondence theory, if as a matter of fact there exist in the real world three entities, A, B, and C, the first of which *is* between the others. The same proposition is true, according to the coherence theory, if it is compatible with all other true propositions. While the correspondence theory and the coherence theory have frequently been set over against each other as rival theories of truth, they do not seem to me to be opposed. They are rather supplementary. The former is

[1] *Essay Concerning Human Understanding,* Book IV, Chap. I.

a theory of the *meaning* of truth; the latter of the *test* of truth. Ideas and propositions are true if they agree with reality. This is what we *mean* by their truth. On the other hand, the only way we have to assure ourselves that this correspondence with reality exists is to show that the item of knowledge which is in question coheres with other knowledge. Thus the rationalistic theory of the test of truth is to be identified with the coherence theory. This is the theory which is presupposed by what we called in the preceding chapter the "method of conceptual analysis," and the "method of hypotheses."

Of course, we can never be absolutely sure that coherence is an index of correspondence with reality. And the skeptic is at liberty to make what he can of this admission. We can only postulate that the agreement of our ideas with one another is an evidence of their truth. This may be taken as a generalization of the account of postulation given in the preceding chapter.

Authoritarianism.—Thinking is hard work, and it is not surprising that there should be a widespread disposition to lean upon some external authority. This has been especially evident in the case of theological inquiries. Thus Protestant Christians have been wont to prove the truth of their tenets by showing their conformity with the Scriptures; Roman Catholics are accustomed to appeal, not only to the Scriptures, but also to the decisions of the councils and the traditions of the Church. Among Protestants different opinions have been held of the degree of importance to be attached to the historic creeds, and not all regard the Bible as, in the literal sense of the term, *infallible,* i.e., completely free from error. Roman Catholics, on the other hand, after centuries of discussion, came to the conclusion in 1870 that the teaching authority of the Church is concentrated in the Pope of Rome, who is the responsible interpreter both of

Scripture and tradition. And when the Pope speaks *ex cathedra,* that is to say, "from the chair," or officially, he is said to be divinely safeguarded against error.

The Partial Truth of Authoritarianism.—As long as the believer in infallibility, whether of Pope or Bible, is content to *postulate* the trustworthiness of his favorite source of doctrine, he is invincible. In other words, if he has the hardihood to renounce the authority of reason, and if he then adheres consistently to this renunciation, his position is unassailable. But this invincibility he shares, as we have seen, with the skeptics. He also shares it with the devotees of rival infallibilities. And we may be pardoned for declining to take the infallible word of any infallible authority as sufficient evidence for its own infallibility. If, however, the infallibilist appeals to reason, he gets into a curious position. For he thereby admits the authority of the logical test of truth. And, if logic may be trusted to vouch for the correctness of the proof of infallibility, it can also be trusted to pass judgment upon specific doctrines.

The residuum of truth in the authoritarian theory is the genuine insight that the discovery of truth is a coöperative enterprise. It is impossible for any one mind to subject everything to the tests prescribed by logic. Just as we must postulate the general trustworthiness of our own faculties if we would think at all, so we must assume the essential soundness of the mental processes of others, if we would be assisted by them in the work of discovery. In regard to matters lying beyond the province to which one has devoted special attention, one necessarily relies upon those who may be considered *experts* in the particular field in question. In each case, however, the right to speak with authority must be *earned.* It is only as a man, a book, or an institution has been found trustworthy that he, or it, may rightly expect to be trusted. Every investigator should, then, think of him-

self as a member of a group of workers; and the discovery of truth as the work of the group rather than of the individual. Whatever may have been possible three or four centuries ago, it would now be absurd for even the greatest genius to attempt to play a "lone hand." But the individual investigator ought not to defer to the established opinion of his group. Neither should the group seek to establish an opinion. The right to criticize our authorities must be insisted upon. We should not treat any authority as infallible. Indeed, criticism is involved in the process by which the prestige of any given authority is to be built up. In order that the coöperative enterprise of truth-seeking may succeed in the fullest measure, each individual must make his contribution. Everyone ought to choose some field in which he can reasonably hope to become a master; and in that field, at least, he should dare to think independently. The results which he reaches may be erroneous; but the world has a right to know what they are. And anyone who, whether from false humility or some other motive, covers up or extinguishes the light which is in him, becomes thereby a traitor to the cause of truth.

The Claim of the Mystic.—The authoritarian is content to accept as true such doctrines as he finds in his canonical books or in the official deliverances of the institution to which he belongs. The rationalist insists upon the right to test all things for himself in the cold, dry light of reason. A third position is that of the mystic, who lays claim to an immediate acquaintance with ultimate reality.

In order to understand what is meant by "mysticism," it may be helpful to begin with the conventional distinction between two kinds of knowledge. These are sense-perception and reasoning. In sense-perception we come into immediate touch with reality, while in reasoning or inference our knowledge of reality is indirect. Psychological inquiry, it

is true, tends to break down the hard and fast distinction between these two kinds of knowledge. Sense-perception is found to be less direct and immediate than it seems. No sharp lines can be drawn; the difference between reason and perception is one of degree rather than of kind. Yet the distinction between reason and sense, between mediate and immediate knowledge, possesses *prima facie* validity.

Now it has been customary among those who distinguish these two kinds of knowledge to regard "reason" as higher than "sense," that is to say, to attribute to it a greater worth or dignity. A further step is taken by the mystic. He holds that there is a third kind of knowledge, which is as far above reason as reason is above sense. This third kind of knowledge,—which, perhaps should not be called knowledge—possesses all the immediacy of sense-perception, and yet is immeasurably superior to sense-knowledge.

The Nature of the Mystic Experience.—Not many in any generation have laid claim to this mystic illumination, and even those who have laid claim to it have not been in perpetual possession of it. For example, Plotinus (about 250 A. D.) is said to have enjoyed the mystic ecstasy but thrice in his life-time. Moreover, the mystic experience is usually described as "ineffable," i.e., as incapable of utterance. The mystic, returned from his ecstatic experience, is unable, for the most part, to tell what he has seen. He knows that it has been an extraordinarily precious experience; but just what it has been, or just what he has learned, he cannot tell.

It is obvious that in so far as the mystic experience is ineffable it cannot be made a test of truth. And yet we must not be too hasty in our condemnation. The fact that the mystic is unable to give a satisfactory description of his experience is not a sufficient proof that his insight, so long

as it lasted, was not genuine; or even that his broken utterances concerning it are wholly without value. For supposing the mystic's illumination to be veridical, it would nevertheless be exceedingly difficult for him to tell the rest of us about it. His case would be like that of a man with eyes like ours, who should be born in a race of blind men. If he tried to tell about his visions of color and brightness the rest would in all probability consider him insane. His experience, too, would be ineffable. And even in trying to make clear to himself what he had seen, such a one would lack words in which to describe it; for language is a social product, and his language would contain no terms in which to describe visual experience.

Mystic Experience Pathological.—It may be objected, however, that the experience of the mystic is pathological, and therefore not genuinely objective. And it must be admitted that there is a great deal in the lives of noted mystics which supports the pathological theory. To mention only a few facts which look in this direction, the mystics have usually been members of monastic orders. They have given themselves to ascetic practices. In their effort to merit sainthood they have for long periods gone without food. They have spent in their devotions long hours which nature would claim for sleep. They have been voluntary celibates, keenly conscious of the urge which they were resisting. Now there is reason to believe that fasting and vigils have a tendency to produce phenomena similar to the experiences of the mystics; and many students of the subject believe that the mystic experience is, at least in part, a sex phenomenon. Nevertheless the inference from the pathological nature of the mystic experience to its hallucinatory character is a *non sequitur*. Even if the morbity of the mystic ecstasy be granted, it does not necessarily follow that it is non-veridical. Indeed, it is not impossible that the so-called delusions of the insane re-

veal aspects of reality which are hidden from those minds which we call "normal."

The Prima Facie Truth-Claim of the Mystic Experience.—Does "mystic knowledge," then, have any claim to consideration? Our answer to this question is that such putative knowledge has exactly the same claim to consideration—no more and no less—as any other alleged cognitive experience. While we concede the equal *prima facie* truth-claim of the mystic experience, we cannot grant it a privileged position. In other words, it cannot justly demand immunity from the test of truth to which all our other cognitive experiences are subjected. The most that can be said for mysticism is that it *may* have a unique *source* of knowledge. It does not give us an additional *criterion*. It is sheer assumption to suppose that an immediately apprehended datum is more likely to be veridical than the conclusion of a process of reasoning. So far as the mystic experience is cognitive it must stand on the same plane as the data of sense-perception. It is only by virtue of the principle of coherence that we are able to distinguish between genuine percepts and illusions or hallucinations. Any bit of sensory content which does not conform to our organized world of experience is pronounced unreal. Any percept which does not harmonize with the percepts which already have possession of the field is called an illusion. In the same way, such mystic apprehensions or insights as harmonize with the rest of our experience may be accepted as veridical, while such as do not must be rejected. In other words, the test of truth remains the convergence of the evidence in favor of a hypothesis.

Intuitionism.—In our discussion of mysticism we have anticipated nearly all that need be said about intuitionism. Intuitionism is the theory that there are certain truths which are self-evident. Self-evidence thus becomes a criterion of

truth. For, so far as these truths are concerned, no other test is required. The difficulty with intuition is, of course, that self-evidence is not a sufficient criterion. Propositions may be self-evident and yet be untrue. What is self-evident to me now may not be self-evident to someone else, or even to me at some other time. In thinking of intuitions, it is well to bear in mind F. C. S. Schiller's remark that "geniuses, ladies, and lunatics" are "particularly prone to them." [1] If, then, we wish to follow intuition, the question naturally arises, Whose intuition shall I follow? Mine or that of the person next door? Mine to-day, or mine twenty years ago? If the deliverances of intuition are incompatible, they cannot all be true. Some must be taken and others left. Of course, I may arbitrarily decide to choose mine instead of yours, and mine now instead of mine at some other time. But if we are not content with an arbitrary decision of this kind, the only principle which remains for our guidance is that of coherence. Such "self-evident truths" as can be harmonized with our world of organized knowledge may be accepted. Such "self-evident truths," on the other hand, as cannot be harmonized with our wider system of knowledge, in spite of their boasted self-evidence, must be cast aside. In other words, not self-evidence, but *organizableness,* is the ultimate criterion of truth.

Pragmatism.—Authoritarianism, mysticism, and intuitionism may all be classified as "anti-intellectualist" positions. They agree in disparaging the claims of human reason. Pragmatism, too, has usually been regarded, both by its opponents and by its defenders, as an anti-intellectualist doctrine; but, as we shall see, it may be so interpreted as to differ little, if at all, from rationalism. The name "pragmatism" was given by William James to the "attitude of looking away from first things, principles, 'categories,'

[1] *Formal Logic,* p. 226.

supposed necessities; and of looking toward last things, fruits, consequences, facts." [1] As a theory of the test of truth—which is the only aspect of it with which we are now concerned—pragmatism, accordingly, insists that the test shall be *practical.* Doctrines are to be judged by their *fruits* rather than by their *roots.* "Ideas become true," says James, "just in so far as they help us to get into satisfactory relations with other parts of our experience." [2] "The true . . . is only the expedient in the way of our thinking, just as 'the right' is only the expedient in the way of our behaving." [3] Or in Schiller's phrase, which is taken over by James, the true is that which "works."

In so far as pragmatism is a protest against the notion that truth is something "transcendent," or "absolute," by which James appears to mean *superhuman,* it has our sympathy; for if truth were superhuman, we should be tempted to resort to extra-logical modes of judging it, such as the mystic and the authoritarian. A difficulty arises, however, when we inquire just what values are to be taken into account in deciding whether a doctrine "works." In other words, what are the satisfactions which justify belief? James himself, it must be confessed, is not always clear or unambiguous on this point. In his more careful statements, he gives the precedence to intellectual values. Logical considerations, he tells us, ought to prevail. In less careful moments, on the other hand, James seem to hold that other values, æsthetic or religious, may be allowed to tip the scale. As Professor Sellars remarks, "There runs through James the suggestion of another test of truth than that worked out by science and logic. It is not prominent, but it is present enough to have led many to interpret his position in accord-

[1] *Pragmatism,* p. 54f.
[2] *Ibid.,* p. 58.
[3] *Ibid.,* p. 222.

ance with it. This second standard consists in an appeal to personal feelings and satisfactions as a foundation for truth." [1] And, I do not think it is unfair to say, it was this suggestion, contrary as it was to James's real meaning, which gave pragmatism its vogue. The authority of William James was quoted for the thesis that doctrines which could not be justified scientifically might nevertheless be believed *if they worked;* and if a given doctrine produced a satisfactory emotional response in the believer, or tended to produce fruits of right living, it was said "to work." Even doctrines which seemed positively repugnant to reason were held by some self-styled pragmatists to be worthy of credence, if they were practically helpful or emotionally precious.

In opposition to this interpretation of pragmatism, which was a perversion of James's position, we must insist that a proposition may be useful, precious, soul-satisfying, and still be false. There are, obviously, fruitful errors and harmful truths. Religious and æsthetic values may, perchance, be more important than logical values; worship and appreciation may be *better* than knowledge; nevertheless, whether or not they are better or more important, the one set of values cannot rightly be substituted for the other. Truth cannot be established by an appeal to æsthetic and religious satisfactions. Such an appeal is always an *ignoratio elenchi.*

If, however, the satisfactions which are to be considered in deciding whether or not a proposition "works" are logical satisfactions then the pragmatist's account of the matter becomes an excellent statement of the way in which alleged truths are to be verified. Thus interpreted, pragmatism is indistinguishable from what we have called rationalism. To say that "the truth is that which works, that which gives a maximum of satisfaction," is then but a restatement of the principle employed by the natural sciences in testing their hy-

[1] Sellars, *Essentials of Logic,* p. 308.

potheses. And, as I have said, in his more careful moments, James states clearly enough that this is his real position. In making decisions concerning ultimate metaphysical and religious issues, the basis of choice should be the same as in weighing the merits of rival scientific theories, viz., the relatively successful "working" of the hypotheses in question. "The point I now urge you to observe," says James, "is the part played by the older truths. Failure to take account of it is the source of much of the unjust criticism leveled against pragmatism. Their influence is absolutely controlling. Loyalty to them is the first principle—in most cases it is the only principle. . . . New truth is always a go-between, a smoother over of transitions. It marries old opinion to new fact so as to show a minimum of jolt, a maximum of continuity. We hold a theory true just in proportion to its success in solving this 'problem of maxima and minima.' " [1]

Accordingly, when pragmatism, asking "its usual question," inquires, "What, in short, is the truth's cash-value in experiential terms?" [2] and, answering its own question, says, "True ideas are those that we can assimilate, validate, corroborate, and verify," [3] we must not be misled by the crassly utilitarian associations of the phrase "cash-value." To *assimilate,* to *validate,* to *corroborate,* to *verify,*—all of these denote an intellectual rather than an emotional process. "One of the accusations which I oftenest have to meet," protests James in his preface to the *Meaning of Truth,* "is that of making the truth of our religious beliefs consist in their 'feeling good' to us, and in nothing else." This, of course, was never his intention.

Coherence the Test of Truth.—Our brief study of pragmatism, accordingly, brings us back again to what is

[1] *Pragmatism,* p. 61.
[2] *Ibid.,* p. 200.
[3] *Ibid.,* p. 201.

essentially the rationalist position. An idea is true if it fits in with the rest of our ideas. A proposition is true if it harmonizes with other propositions. A new idea is like a candidate for admission to a club or a fraternity. It gets in if it is not obnoxious to any of the present members of the organization of ideas.

Sometimes, however, in order to make room for new ideas the antagonistic old ideas are expelled. When the old and the new are incompatible, the former may shut out the latter or the latter may succeed in displacing the former. And thus the discovery and revision of knowledge goes on. We make our choice between the new and the old, either repelling the new or expelling the old, so as to obtain the most satisfactory adjustment. We believe as much as we can. We so organize the multitudinous items of our experience as to reduce to a minimum the number of items which must be rejected.

Summary.—The problem of the test of truth should not be confused with that of the source of truth or with that of the meaning of truth. According to rationalism the truth of a proposition is evidenced by its coherence with all other true propositions. Authoritarianism holds that the true doctrine is that which agrees with the pronouncements of accredited authority. Mysticism and intuitionism agree in appealing to the self-evidence of immediate experience. Pragmatism judges an idea by the satisfactoriness or unsatisfactoriness of the experiences to which it leads.

Exercises.—1. For what kind of knowledge must we depend on external authority?

2. In deciding what is true should we consider our wishes?

3. Discuss the relative trustworthiness of intuition and reason.

4. Do you know of any man, any book, or any institution which is infallible?

5. If two propositions are incompatible, how can you tell which one to believe?

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TABLE OF POSTULATES

The following postulates are suggested as necessary and sufficient for the exposition of the deductive logic *"more geometrico."*

1. The principle of the modus ponens: If *m* is *n* implies *x* is *y*, and *m* is *n* is affirmed, then *x* is *y* must also be affirmed.

2. The principle of the modus tollens: If *m* is *n* implies *x* is *y*, and *x* is *y* is denied, then *m* is *n* must also be denied.

3. The principle of identity: *x* implies *x*.

4. The principle of contradiction: *x* and *non-x* cannot both be true.

5. The principle of excluded middle: *x* and *non-x* cannot both be false.

6. The principle of double negation: If the negative of *x* is *non-x*, the negative of *non-x* is *x*.

7. The principle of subalternation: All *a* is *b* implies Some *a* is *b*. (This is the relation of A to I. If we now *define* E and O as the contradictories, respectively, of I and A, we may derive a similar relation of E to O.)

8. The reverse principle of subalternation: Some *a* is *b* does not imply All *a* is *b*.

9. The principle of the denial of the antecedent: If *m* is *n* implies *x* is *y*, and *m* is *n* is denied, *x* is *y* is undetermined.

10. The principle of the affirmation of the consequent: If *m* is *n* implies *x* is *y*, and *x* is *y* is affirmed, *m* is *n* is undetermined.

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